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Reprint of: A new simplified analytical model for soil penetration analysis of rigid projectiles using the Riemann problem solution☆,☆☆

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ABSTRACT

A new simplified analytical model to analyze the penetration of rigid projectiles into soil media is presented. The soil medium is represented by a set of discs, responding in the radial direction under plain strain conditions. A convenient mathematical formulation is derived based on some simplifying assumptions. According to the present approach, the contact parameters in each disc are computed using the developed exact solution of the symmetrical Riemann problem for an irreversible compressible medium. One of the new features of the present model is the incorporation of the exact nonlinear equation of state including unloading-reloading thus considering another key variable that is the maximum medium density that is attained in the process of active loading before unloading is started. Thus the new model considers unloading in the soil medium during the progress of penetration. The present model focuses on the projectile motion and provides information on its velocity, deceleration and depth time histories. It also provides information on the interaction of the projectile with the surrounding soil such as the normal stress distribution along the projectile nose and is capable of determining the contact zone between the nose and the soil. Comparison of the present model results with two-dimensional numerical results as well as with different analytical models and with experimental data is performed. The present model predictions are found to be in good agreement with test data and superior to many existing simplified models. Contrary to many other simplified models, the present model is purely theoretical and does not require any empirical constants or special arbitrary assumptions for calculation of the contact pressures along the projectile nose at all times. The calculations require very small computer time and provide much information regarding the projectile motion in the soil medium.

1. Introduction

The penetration of projectiles into soil media has been a subject of research for many years. The purpose of analytical and semi-analytical models is to enable economic and fast evaluation of projectile and soil response during the penetration event that will be in good agreement with experimental data and will supply the most important information on the projectile's motion in the medium (velocity, deceleration and penetration depth time histories) that may be obtained by 2-D wave propagation computer programs.

A large number of empirical formulae and engineering models have been proposed over the years, as reviewed in many review papers and books such as Backman and Goldsmith [1], Rosenberg and Dekel [2], Ben-Dor et al. [3], etc.

Therefore, we can refrain from a detailed literature review on this matter, and may focus on the subfield of the present paper only that is analytical models for penetration analysis.

Existing analytical models are mostly based on the spherical or the cylindrical cavity expansion solutions. The formulation is based on the conservation of momentum, mass and energy equations that together with the constitutive equations and boundary conditions, including at the shock front discontinuity, constitutes the set of equations that is required to solve the problem. These are simplified 1-D solutions that follow the expansion of a spherical or a cylindrical cavity which is very different from the cavity formed during penetration. Cavity expansion was first introduced by the works of Bishop et al. [4] and Hopkins [5], but its application to penetration into soil, rock and concrete was mainly done by Forrestal et al, who adopted the spherical cavity expansion model to analyze ogive nose projectile penetration. To implement the cavity expansion model to penetration problems it is required to determine the relationship between the radial pressure acting on the cavity boundary and the radial internal cavity boundary motion parameters (displacement, velocity and acceleration) and relate them to the geometry and

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motion of the penetrating projectile. Commonly a full contact is assumed between the projectile nose and the target. A constant pressure distribution along the nose is assumed that equals to the cavity boundary pressure. Based on these assumptions and the fundamental cavity expansion solution, the distributed pressure can be integrated over the nose surface area to determine the instantaneous resistive force, with which the equation of motion is solved to yield the projectile motion time history.

In their paper on modeling penetration in soil Forrestal and Luk [6] idealized the soil pressure–volumetric strain relationship by a locking solid with a predefined constant volumetric strain η^*. They considered the shear-strength-pressure behavior as a Tresca or a Mohr-Coulomb or a Mohr-Coulomb with Tresca-limit yield criteria.

Forrestal and Tzou [7] developed a spherical cavity-expansion penetration model for concrete targets. Their modeling could be considered as an extended model for soil penetration analysis as well. They analyzed the problem assuming that the material is either compressible or incompressible. A linear pressure-volumetric strain relationship was assumed and the shear strength-pressure relationship was idealized as Mohr-Coulomb with a tension cutoff.

Yankelevsky et al developed the DISCS model [8–14] in which the target medium is subdivided into small layers of constant thickness (DISCS). The idea behind the DISCS model is that most of the penetration depth in the soil, beyond the shallow crater at the top surface, is of a tunneling type. The induced velocity field in the soil is almost radial, thus the medium response is in the disc's plane. Therefore the discs model with independent discs acting in their planes is justified. This is a simplifying assumption that allows analysis in the discs plane. The DISCS model aims at better representing the projectile-soil interaction, than the spherical or cylindrical cavity expansion models. In the latter there is no connection between the infinitely long cylindrical cavity or the spherical cavity shape to the projectile nose shape. In the DISCS model every disc is interacting with the penetrating projectile nose, and the internal boundary motion characteristics of every disc are determined from this interaction depending on the nose shape and velocity. Although each layer is modeled independently their internal boundary radii along the nose forms a smooth continuous function in the vertical direction that is equal to the nose shape, as long as contact exists. This enables a considerably more realistic description of the cavity developed during penetration that is formed from the local cavities of all discs that are in contact with the projectile nose during its motion.

The DISCS model also improved the constitutive modeling that was commonly used in cavity expansion models, which was either a locking or a linear pressure-volumetric strain relationship. The new material model was a that of a floating locking model, with a variable locking strain that is determined from the real nonlinear equation of state by considering the variable stresses and strains along a disc's radial coordinate and determining an equivalent pressure and a corresponding volumetric strain, that is based on equating the internal work done by the average stresses over components of the average strain increments with the internal work done by the variable stresses over the variable strain increments. That procedure enables a rational selection of a representative simplified locking strain that is tightly connected to the test material properties and to the actual stress level.

The DISCS model provides high quality predictions for high velocity penetration into a thick target and enables various calculations of the projectile time history, the target's response and the interaction pressure distribution at all times. It predicts the interaction condition between each disc and the nose and determines if contact exists and calculates the interaction local pressure.

In this present work a new simplified analytical model is developed that is based on the DISCs model [8–14]. According to the present approach, the soil medium is subdivided into discs of variable thickness (Fig. 1). The response properties of each disc are concentrated at the interior expanding cavity where the interaction with the penetrating nose occurs.

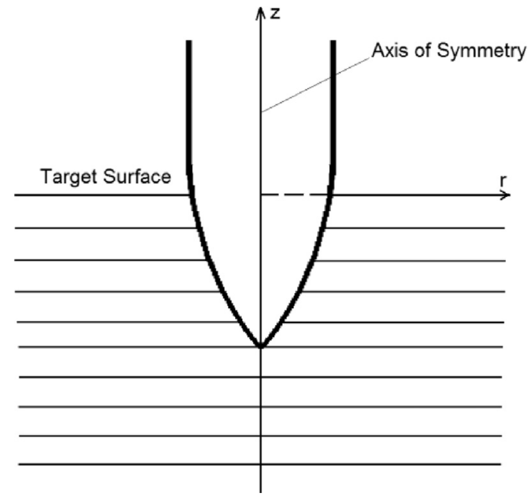

Fig. 1. A continuum idealization with soil discs [8–14].

Although similar to the original DISCS method [8–14] in its geometrical idea, the proposed approach is entirely different and the essence of its new features is as follows:

- Compared to the original DISC model, the constitutive model is further improved. In the DISCS model [8–14], the medium material behavior is described by a “floating” ideally locking material and therefore the soil density distribution along the disc's radial coordinate was constant. In the present model, the locking density ρ_L (Fig. 2a) is not requested (Fig. 2b). The medium is represented with its exact nonlinear equation of state and the contact soil density that is continuously updated according to the soil equation of state and the actual current state of stress at the projectile-soil interface.
- A new important feature that does not exist in other analytical models is the inclusion of the unloading branches in the general formulation of the equation of state. This feature is essential for an appropriate formulation of the penetration problem, as will be described in the following.
- In the original DISCS model [8–14] the axis of symmetry is subdivided into equal finite differences, and the soil medium is divided into discs of a finite difference thickness, by planes being perpendicular to the axis of symmetry. In the present model, the numerical procedure finds an advantage in determining the time steps as the independent parameter and the projectile advancement in every time step determines the disc thickness. Thus, the number of discs is not constant, their thickness is not equal and each new time step evolves a new disc (Fig. 3).
- A major new feature of the present model is the numerical approach. While in the DISCS model the shock wave propagation was calculated in each disc being in contact with the nose, in the present model interest is focused on the common interface between the nose and the disc only and the contact parameters at the common boundary are computed using exact solution of the so-called symmetrical Riemann problem (Fig. 2b). This is an entirely new formulation that has not been used in other penetration models.

It is well known that the Riemann problem for a system of conservation laws of a given medium is defined as the Cauchy problem with initial conditions consisting of two parts of a one dimensional medium, each of them is assigned with different constant parameters that are separated by a discontinuity at the origin. The solution