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Capturing pressure- and rate-dependent behaviour of rocks using a new damage-plasticity model

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Rock response to confining pressure and strain rate can change dramatically from very brittle to ductile. Capturing this transition is crucial for a correct prediction of rock mass failure due to blasting, explosion or drilling in mining. In this work, a new constitutive model that accounts for the effects of both confining pressure and strain rate on the nominal strength and post peak behaviour is proposed for dry intact rocks and other similar geological materials. The key features of the proposed constitutive model are the employment of a single loading function that evolves from initial yielding to ultimate failure during damaging and the rate-dependent enhancement so that the strain rate effects can be faithfully described at different confining pressures. The model can adequately capture both the brittle and ductile responses as well as the brittle-ductile transition as a result of both strain rate and confining pressure.

1. Introduction

Understanding and modelling the behaviour of rocks would pave the way towards a safe and economical design in geotechnical and mining engineering. Like any other geological material, rock behaviour depends on both strain rate and pressure, that is, its response can change from very brittle to ductile under significantly high confining pressure [1–10] or high strain rate [11–19]. In mining and geotechnical operations in crustal rock formations the effects of blasting, drilling or tunnelling and/or tectonic and earthquake forces can dramatically change the stress state. Therefore, the prediction of failure would require a model that can capture, as accurately as possible, the macroscopic response of rocks under rather complex stress states and different loading rates.

Inelastic rock deformation at microscopic scale involves a series of micro-mechanical processes leading to degradation of the material micro-structure. These processes usually begin with initiation of micro-cracks within the material matrix (at low pressures) and/or grains (at high pressures), followed by localization of micro-cracks with a band of certain thickness where micro-cracks finally coalesce to form the macroscopic fracture. Throughout the entire course of inelastic deformation, frictional sliding between the two faces of micro-cracks, asperity interlocking, granular and/or diffusional flow, crystal plasticity, and other processes may accompany micro-cracking and fracturing [20–24]. These micro-mechanisms of deformation are observed at macro-scale as stiffness and strength reduction, inelastic dilation/compaction and residual strains. From a phenomenological perspective, all the mechanisms that cause stiffness and strength reduction may be described as damage and all those phenomena that lead to the occurrence of the residual strains can be interpreted as plastic deformations. Furthermore, the micro-mechanical processes of deformation in rocks are mostly time/rate-dependent, which consequently give rise to the macroscopically observed rate dependent behaviour of rocks. Examples of such time/rate dependent micro-mechanisms may be given as time dependency of static friction and the evolution of frictional strength with the loading rate [25–29] and/or time dependent micro-crack growth [19,30]. Rate dependent macroscopic rock behaviour may be characterized by the increase in the rock strength under tension and compression at high strain rates [31–36]. In addition, at higher strain rates rocks show a tendency towards more ductile behaviour, while in quasi-static loading, under the same confining pressure, the behaviour can be completely brittle [13,14,19,37].

Phenomenological coupled damage-plasticity models with different levels of complexity and applicability have been proposed for

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List of symbols

dt	Time increment
g	Plastic-damage potential
g_f	Local or specific fracture energy
m	Strain rate parameter
p	Hydrostatic stress
p_{c0}, p_{t0}	Pressures at initial yield under isotropic compression and tension, respectively
q	Shear stress
y	Yield function
A, B	Material parameters controlling damage evolution
C_{ijkl}	Secant elastic stiffness tensor
C_{ijkl}^{epd}	Elasto-plastic-damage tangent stiffness tensor
D_{mnkl}	Secant compliance tensor
D	Damage parameter
G	Shear modulus
G_f	Fracture energy
H	Heaviside function of hydrostatic pressure
I_1	First invariant of the stress tensor
J_2	Second invariant of the deviatoric stress tensor
K	Bulk modulus
M_0	Material constant controlling the shape of the yield surface
M_u	Stress ratio at failure
S_{ij}	Deviatoric stress
α_0	Material constant controlling the shape of the yield surface
β	Parameter controlling the dilational response
γ	Material constant controlling the shape of the yield surface
ε_{ij}	Total strain tensor
ε_{ij}^p	Plastic strain tensor
$\varepsilon_s, \varepsilon_v$	Total shear and volumetric strains, respectively
$\varepsilon_s^p, \varepsilon_v^p$	Plastic shear and volumetric strains, respectively
ϵ_p	Accumulated plastic strain
η	Viscosity parameter
λ	Damage-plastic multiplier
σ_{ij}	Stress tensor
$\sigma_1, \sigma_2, \sigma_3$	Major, intermediate and minor principal stresses, respectively
σ_t	Tensile strength
ψ	Elastic energy potential

modelling the mechanical behaviour of rocks [10,38–44]. These models, in general, specify the interaction between damage and plasticity processes in the model formulation in order to account for both stiffness and strength reduction and residual strains, observed during the course of inelastic deformation. Various approaches may be taken for constructing coupled damage-plasticity models for rocks which can reflect the effect of confining pressure on the mechanical behaviour of rock. Examples of such approaches can be given as models with two loading surfaces [39,45–48] or models in which damage evolution is defined as a function of volumetric plastic strain rate [41,49–52]. Although many coupled damage-plasticity models can be found in the literature, which are capable of describing the mechanical behaviour of rocks under different confining pressures, not much attention has been given to constructing models which take into account the combined effects of both confining pressure and loading rate, particularly, on the brittle-to-ductile transition in rocks.

A new formulation is developed in this study by employing a combined yield-failure function, which eliminates the need for multiple loading surfaces, while facilitating both the implementation and applications of the model to cover a wide range of responses under different pressures and strain rates. In this formulation, an initial yield surface evolves, with the evolution of the damage variable, defined as a function of the rate of accumulated plastic strain, into a linear frictional failure envelope [53]. This is an important feature of the proposed model as the brittle/softening and ductile/hardening responses together with the transition between the two states of behaviour, with increasing confining pressure, can be automatically captured by the model without any need for separately defining hardening or softening laws. The fundamental mechanisms which determine the rock behaviour when switching from tensile to compressive loading is also accounted for, through mimicking the influence of micro-crack closure on the rock mechanical response. Furthermore, strain rate dependency of the behaviour, which has been experimentally observed in rocks and other similar geological materials, is incorporated in the model through the Perzyna type viscosity. It should be noted that rate dependency in the proposed model is viewed as an intrinsic feature of the model rather than a regularization technique in numerical implementation. In addition, rate dependent response induced by pore pressures in moist rocks is not taken into account yet in this study, and this is acknowledged as an issue to be addressed in our future work. The separation of pressure and rate dependence in the proposed model formulation allows for independent calibration of the model in quasi-static and dynamic loading conditions. In our opinion, this facilitates both the development and calibration of the model.

The paper is organised as follows; at first the model formulation is described, while providing links between the model formulation and the rock behaviour under different confining pressures and strain rates. Numerical implementation algorithms are then described and numerical examples of rock behaviour under different confining pressures and strain rates are given to demonstrate promising features of the new model.

2. Model formulation

2.1. Convention and definition

In the present study, the tensile stresses are assumed to be positive and the compressive stresses are considered as negative. However, in order to be consistent with the rock mechanics convention, the stress-strain signs will be switched when presenting the model predictions at the material level. The principal stresses are assumed to satisfy the condition $\sigma_1 \geq \sigma_2 \geq \sigma_3$. The first invariant of the stress tensor, σ_{ij}, is denoted as $I_1 = \sigma_{kk}$ and the second invariant of the deviatoric stress tensor, S_{ij}, is $J_2 = \frac{S_{ij}S_{ij}}{2}$, where $s_{ij} = \sigma_{ij} - \sigma_{kk}\delta_{ij}/3$. In addition, two stress invariants, the hydrostatic stress $p = -\sigma_{kk}/3$ and the shear stress $q = \sqrt{3J_2}$ are used in the model formulation.

2.2. Stress-strain relationship

The proposed constitutive model for intact rock has been formulated within the framework of continuum damage mechanics and plasticity theory. In addition to the compressive response, the model accounts for the tensile regime and includes micro-crack closure effects when switching from tension to compression modes. Such effects are indirectly modelled by considering an enhanced elastic stiffness under compressive loading. Instead of introducing an isotropic damage parameter, the unilateral behaviour of rock due to crack closure has been described by coupling the damage parameter D with the elastic stiffness tensor. The following form of the elastic energy potential has been assumed for this purpose:

$$\psi = \frac{1}{2}[(1-D) + DH(-\sigma_{kk})]K(\varepsilon_v - \varepsilon_v^p)^2 + \frac{3}{2}(1-D)G(\varepsilon_s - \varepsilon_s^p)^2 \qquad (1)$$

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