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Essential physics of target inertia in penetration problems missed by cavity expansion models

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ABSTRACT

The problem of a rigid projectile with the shape of an ovoid of Rankine penetrating an incompressible elastic-perfectly-plastic target is used as an example to study the dependence of the drag force F on the penetration velocity V. The phenomenological functional form of the contact pressure P proposed by Hill (1980) during World War II, the analytical solution in Yarin et al. (1995), the numerical simulations in Rosenberg and Dekel (2009), as well as new numerical simulations in this work all consistently reveal the importance of a physical flow field in the target material. Below a critical value V_s of V, the drag force F is constant. The critical value V_s determines the onset of separation of the target material from the projectile's surface. Axial inertia being converted into radial inertia in the target near the projectile's tip controls the physics of the separation process and the strong dependence of F on V for $V > V_s$. Cavity expansion models based on cylindrical or spherical flow fields miss the essential physics of this separation phenomenon and are incorrect when target inertia is important. Also, the numerical simulations indicate that the constant value of the drag force for $V < V_s$ depends on the tip shape, which cannot be accurately predicted by cavity expansion models. Since cavity expansion models cannot accurately predict results of the simplest problem of a rigid projectile penetrating an incompressible elastic-perfectly-plastic target, it should not be assumed that these models are accurate for general target materials (which include compressibility, hardening and porosity), even though the models are simple to use.

1. Introduction

Penetration mechanics has been of interest to engineers for a very long time. Comprehensive reviews of this subject with many references to previous work can be found in Refs. [1,2]. Also, a large amount of experimental data has been collected in Ref. [3]. Specifically, it is noted that static solutions for the pressure required to expand spherical and cylindrical cavities in an infinite ductile material were developed in Ref. [4], and Hopkins [5] developed a dynamic solution for expansion of a spherical cavity into an elastic-plastic material. These analytical solutions assume that the target material is incompressible, and Hill [6] included compressibility for expansion of a spherical shell.

Fig. 1 shows a sketch of a projectile of length L and tail radius R, which is moving in the negative $\mathbf{e}_z$ direction at velocity V. The surface of the projectile is defined by the radial position

$$r = \hat{r}(\xi), \quad \xi = z - z_0(t) - \frac{R}{2}, \quad \hat{r}\left(-\frac{R}{2}\right) = 0, \quad \hat{r}(\xi_L) = R, \tag{1}$$

where z is the axial coordinate relative to a fixed point and the location and velocity of the projectile's tip are given by

$$z = z_0(t), \quad \xi = -\frac{R}{2}, \quad V = -\dot{z}_0, \tag{2}$$

with a superposed (•) denoting time differentiation. Also, ξ_s characterizes the axial location of the circle where the target material separates from the projectile's surface and ξ_L locates the projectile's tail. Hill [7] described research on penetration mechanics that was conducted during World War II. In particular, this work focused on the relationship between the shape of the projectile's tip and the cavitation phenomena when the target material separates from the projectile's surface closer to its tip than to its tail, causing the radius of the cavity to be larger than that of projectile's tail.

Specifically, Hill [7] proposed a phenomenological expression for the contact pressure P applied by the target on the projectile's surface of the form

$$P = -(\mathbf{n} \cdot \mathbf{T}\mathbf{n}) = P_1 Y + P_2 \rho V^2, \tag{3}$$

where $\{P_1, P_2\}$ are given by

$$P_1 = \text{constant}, \quad P_2 = k \frac{d}{d\xi}\left[\hat{r}(\xi) \frac{d\hat{r}}{d\xi}\right]. \tag{4}$$

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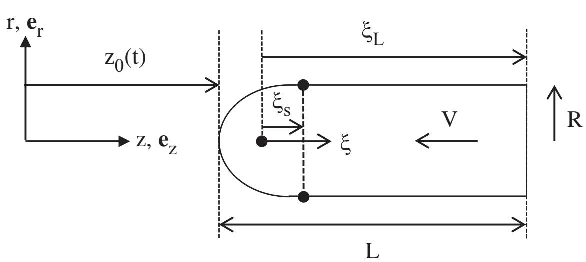

Fig. 1. Sketch of a projectile of length L and tail radius R which is moving in the negative $\mathbf{e}_z$ direction at velocity V. The values $\{\xi_s, \xi_L\}$ denote the axial locations of the circles where the target material separates from the projectile's surface and the projectile's tail, respectively.

In these expressions, P_1 is a positive dimensionless constant controlling the static resistance to penetration, Y is the constant uniaxial yield stress in the target, k is a positive dimensionless constant and ρ is the density of the target. Both of the constants $\{P_1, k\}$ have to be determined empirically. Also, **n** is the unit outward normal to the projectile's surface and **T** is the stress tensor in the target. Using these expressions, the drag force F applied by the target on the projectile in the positive $\mathbf{e}_z$ direction due to Eq. (3) is given by (e.g., Ref. [8])

$$F = -\mathbf{e}_z \cdot 2\pi\int_{-R/2}^{\xi_s} P\,\mathbf{n}\,d\xi = 2\pi\int_{-R/2}^{\xi_s} P\,\hat{r}(\xi)\frac{d\hat{r}}{d\xi}d\xi = F_1 + F_2, \tag{5}$$

with the functions $\{F_1, F_2\}$ having the forms

$$F_1 = \pi\hat{r}^2(\xi_s)P_1 Y, \quad F_2 = \pi k\left[\hat{r}(\xi_s)\frac{d\hat{r}}{d\xi}(\xi_s)\right]^2 \rho V^2, \tag{6}$$

In these expressions, use has been made of Hill's [7] assumptions that the term $\{\hat{r}(\xi)d\hat{r}(\xi)/d\xi\}$ vanishes at the projectile's tip and tail

$$\hat{r}\frac{d\hat{r}}{d\xi} = 0 \quad \text{for } \xi = -\frac{R}{2}, \quad \hat{r}\frac{d\hat{r}}{d\xi} = 0 \quad \text{for } \xi = \xi_L. \tag{7a,b}$$

In particular, it is noted that if separation occurs at the projectile's tail then

$$F = F_1(\xi_L) = \pi R^2 P_1 Y, \quad F_2(\xi_L) = 0, \tag{8}$$

which indicates that the drag force is constant as long as the target material remains in contact with the projectile's surface until it separates from its tail.

Hill [7] assumed that the target material separates from the projectile's surface at the location of the projectile's bourrelet when the penetration velocity V attains the critical value V_s. In particular, he developed an expression [7, eq. 2.4] for the value of V_s which depended on an empirical constant k/n, with n characterizing the curvature of the projectile's surface at the separation curve. Moreover, he stated that as V increases beyond V_s the location of this separation moves progressively away from the bourrelet (*towards the projectile's tip*). Since the value of F_2 in Eq. (6) is non-negative, it follows that the drag force transitions from being constant for $V < V_s$ to being an increasing function of velocity for $V > V_s$. Thus, the phenomenological forms (3) and (4) for contact pressure proposed by Hill [7] qualitatively predict the physical relationship between target inertia, separation and the dependence of the drag force on penetration velocity. It is also noted that Hill [7] studied projectile's with conical, spherical and ogival tip shapes and concluded that separation should occur at all velocities V for projectiles with spherical and conical shaped tips and V_s is finite for ogival shaped tips. In contrast, the numerical simulations in Ref. [9] show that V_s is positive also for spherical and conical shaped tips.

Hill [7] stated that the effect of the projectile's shape has a negligible influence on the static resistance P_1 to penetration and suggested that this value could be estimated using the theoretical predictions [4] for the pressure required to statically expand cylindrical and spherical cavities in a large elastic-plastic solid. However, the numerical simulations discussed in Ref. [9] show that the value of the drag force F_1 in Eq. (8) actually depends on the shape of the projectile's tip. Also, the theoretical analysis in Ref. [8] shows that P_1 depends on the shape of the projectile.

Goodier [10] used the expression for the pressure to dynamically expand a spherical cavity as an approximation of the contact pressure P applied to the surface of a sphere penetrating a ductile material. The problem of an expanding spherical cavity into an infinite media is attractive to theoreticians because the solution depends on only one space variable and time. This theoretical structure has been used to formulate the solution for elastic-plastic targets which exhibit a range of material response including compressibility, strain hardening, rate-sensitivity, pressure sensitivity and porosity (e.g., Refs. [11–17]).

The main idea of cavity expansion models for penetration mechanics is to use the solution of the pressure for steady expansion of a cavity in an elastic-plastic media as an approximation of the contact pressure applied to a projectile penetrating a target. In its simplest form, using the solution for steady-state spherical cavity expansion into an incompressible elastic perfectly plastic media [5,12], the proposed contact pressure P_{SC} applied to the surface of a general shaped projectile is given by (e.g., Ref. [8])

$$P_{SC} = \frac{2Y}{3}\left[1 + \ln\left(\frac{2G}{Y}\right)\right] + \frac{3}{2}\rho v_n^2, \quad v_n = V\left[\frac{\left(\frac{d\hat{r}}{d\xi}\right)^2}{1 + \left(\frac{d\hat{r}}{d\xi}\right)^2}\right]^{1/2}, \tag{9}$$

where v_n is the normal component of the axial velocity V at the projectile's surface.

Another approach to the modeling of penetration mechanics is based on flow fields in fluid mechanics (e.g., Refs. [18–22]) and has been applied to eroding long rod penetrators. A more recent example of using a realistic flow field in the target can be found in Ref. [23] where normal penetration of a rigid projectile of the shape of an ovoid of Rankine into an elastic-perfectly-plastic target was analyzed. Using this approach, oblique penetration was also considered in Ref. [24]. The influence of separation on the drag force applied to the projectile obtained in the solution of normal penetration in Ref. [23] was further analyzed in Refs. [8,25]. In particular, this analytical solution was shown to predict results similar to those suggested by Hill [7].

Still another approach is to develop physical insight about penetration mechanics through analysis of the results of numerical simulations. In particular, mention is made of the work in Refs. [9,26,27]. The analyses in these works show that below a critical penetration velocity V_s the drag force applied by the target on the projectile is constant. Also, as mentioned before, these simulations show that the value of the drag force in Eq. (8) as well as the value of the separation velocity V_s both depend on the shape of the projectile's tip. In addition, it is noted that the experiments using steel projectiles with different tip shapes impacting aluminum targets (e.g., Ref. [11]) were limited to velocities below the critical value for separation V_s. Consequently, separation was not observed in these experiments, as expected.

In the analysis of an ovoid of Rankine, it was shown in Ref. [8] that the quantity P_1 in Eq. (3) depends on the shape of the projectile $P_1 = P_1(\xi)$. Also, it was shown there that when $V = V_s$, the separation circle jumps from the projectile's tail to a location $\xi = \xi_s$ closer to its tip for which