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# A penetration model for metallic targets based on experimental data

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ABSTRACT

The database for depth of penetration by projectiles into semi-infinite targets is quite extensive. The objective is to use the experimental semi-infinite penetration data to predict finite-thickness target effects. Similitude considerations are used to represent penetration response as a function of a normalized impact velocity, which is the ratio of the penetration pressure to the target strength. Then, a least-squares regression analysis methodology was applied to the normalized experimental data to provide an analytic expression for the normalized depth of penetration as a function of the normalized impact velocity over a very large velocity range, typically a few hundred meters per second to over 3500 m/s. We use the analytic expression, along with some simplifying assumptions, to estimate ballistic limit thickness $T_{50}$ and/or the ballistic limit velocity $V_{50}$. If a target is overmatched, i.e., perforated by the projectile, an estimate is also made for the projectile residual velocity and residual length. The experimental data on which the regression fit is based consists of *L*/*D* 10 projectiles, so a procedure is also developed to account for different projectile aspect ratios. A series of examples are provided to demonstrate the utility and accuracy of the approach.



## 1. Introduction

Correlations to ballistic experimental data have existed since the 1800's and possibly earlier. The equation developed by de Marre is one of the earliest (see Meyers [1]). de Marre equated the kinetic energy of the projectile (cannonballs) to a target limit thickness using three empirical factors. One factor is a multiplicative constant and the other two factors appear as exponents on the projectile diameter and the target thickness. The empirical factors had to be determined for every projectile/target combination of interest.

The Bernoulli equation was applied to shaped-charge jet penetration [2]. The theory—based on incompressible hydrodynamic steady-state flow—predicts penetration velocity and depth of penetration as a function of the densities of the jet and target materials. Later, Eichelberger [3] modified the Bernoulli equation to account for target strength effects, specifically for low jet speeds. This target resistance term was calculated as the difference between target and projectile strengths, each of which were estimated to be one to three times the static uniaxial yield stress. Allen and Rogers [4] used this concept of target resistance, which they call the dynamic yield strength, to adjust the hydrodynamic depth of penetration. Allen and Rogers found that the dynamic yield strength was a function of the penetration velocity, where the penetration velocity was estimated from the experimental depth of penetration and hydrodynamic theory.

Tate [5], and independently Alekseevskii [6], modified the Bernoulli equation to account for projectile strength and target resistance, and to account for deceleration of the projectile. It is the Tate–Alekseevskii model that is often cited as the first practical model for eroding penetrators with projectile and target "strength". The target strength is more appropriately referred to as the target resistance since it depends upon strength as well as confinement. The difficulty in defining the projectile and target strengths is that they are not static properties. Tate attempted to account for the dynamic yield strength of the projectile [7] by using an estimate from Recht [8], and estimated the target resistance from cavity expansion theory [7,9]. A summary discussion on projectile strength and target resistance is provided in Ref. [10]. It is possible to get excellent agreement with the Tate model for a limited range of impact velocities, but the model suffers from only using a single target resistance value. In practice, it often appears necessary to let the target resistance change with velocity [11].

Winters [12] reviewed a large number of regressions and analytical models. He concludes that there were insufficient data to

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**Nomenclature**

| | |
|---|---|
| $a, b, c, d, m, r, \omega, \beta, \gamma$ | curve fit parameters |
| $a^*$ | Eq. (28) |
| $A$ | projectile presented area |
| $c_p$ | projectile sound speed |
| $D$ | projectile diameter |
| $D_c$ | crater diameter |
| $k_1$, $k_2$ | proportionality constants |
| $K$ | bulk modulus |
| $\ell$ | instantaneous projectile length |
| $L$ | initial projectile length |
| $L_r$ | residual projectile length |
| $M$, $M_s$ | projectile mass |
| $P$ | depth of penetration |
| $t$ | time |
| $T$ | target thickness |
| $T_{50}$ | ballistic limit thickness |
| $u$ | penetration velocity |
| v | projectile tail velocity |
| $V$, $V_s$ | impact velocity |
| $V_r$ | residual velocity |
| $V_{50}$ | ballistic limit velocity |
| $\overline{V}$ | Eq. (11) |
| $\alpha$ | $\sqrt{\rho_p/\rho_t}$ |
| $\varepsilon_u$ | failure strain |
| $E$ | plastic work/volume |
| $\lambda$ | scale size |
| $\rho$ | density |
| $\xi$ | $L/D$ ratio in Eq. (27) |
| $\Psi$ | Eq. (38) |
| $\sigma$ | dynamic flow stress |
| $\Sigma_Y$ | yield stress |
| $\Sigma_u$ | ultimate stress (at failure) |

*Subscripts*

| | |
|---|---|
| $p$ | projectile |
| $t$ | target |

draw definite conclusions regarding the importance of target and projectile parameters and states that "Empirical formulae have been produced which fit experimental data over limited ranges of materials and velocities." A study by de Rosset and D'Amico [13] analyzed a data set consisting of tungsten rods impacting rolled homogeneous armor (RHA). They opted to ignore projectile and target strengths or hardnesses, noting that "this effect would have to be included in future refinements of the model, assuming that there is enough data in existence…." The data set used by de Rosset and D'Amico included approximately 120 points.

In an earlier program called Project Thor [14], researchers attempted to correlate the mass and velocity of fragments with damage to aircraft components. The initial project involved the detonation of high explosive shells at specific locations about 152 B-25 aircraft, resulting in the eventual analysis of approximately 70,000 fragment strikes. While the fragment materials were somewhat limited, the variation in fragment size and target materials was quite extensive. This led to the creation of what are known as Thor Equations, considered a semi-empirical model for the penetration of plates. Despite a number of limitations, the Thor equations are still used in many survivability/vulnerability analysis tools. There are several equations for estimating parameters such as residual velocity or residual mass. The equations generally have at least five constants that must be determined for each combination of projectile and target materials. The Thor equation for the residual velocity (or limit thickness if rearranged) of normally impacting fragments is given by:

$$V_r = V_s - 10^a (TA)^b M_s^c V_s^d \tag{1}$$

where $a$, $b$, $c$, and $d$ are found by conducting regression analyses on the data.

Eq. (1) contains four fitted parameters; if target obliquity is included, the number of parameters increases. There are several things to note. First, this work began in the 1940's and the form of the equation was selected because it could be handled with the computational power available. Second, regressions were valid only over relatively narrow ranges of projectile mass, striking velocity, and target thickness. Third, virtually all of the historic semi-empirical formulations resulted in parameters with units of length, mass, area, velocity, and possibly others being raised to some power. Such relationships are often a clue that the problem has not been stated in the most general manner.

There have been many other attempts to provide empirical correlations with experimental data. Backman and Goldsmith [15] provide an extensive review of the literature. A somewhat more recent attempt includes more variables related to geometric and material properties, summarized by Wright [16]:

$$\frac{\rho_p V_{50}^2}{E_p}\frac{L}{D} = c\left(\frac{T\sec\theta}{D}\right)^{\omega}\left(\frac{E_p}{E_t}\right)^{\beta}\left(\frac{\rho_p}{\rho_t}\right)^{\gamma} \tag{2}$$

$$E = \frac{1}{2}(\Sigma_Y + \Sigma_u)\varepsilon_u \tag{3}$$

The $T\sec\theta$ term is the line-of-sight thickness of the target, and $E_p$ and $E_t$ represent the energy per unit volume that can be absorbed in the projectile and target materials before failure. The coefficient $c$ and the exponents $\omega$, $\beta$, and $\gamma$ are determined by multiple regression analysis. In contrast to Eq. (1), there is an attempt to structure the empirical expressions in Eqs. (2) and (3) with important geometric and material properties in a nondimensional form so that the regression coefficients do not depend upon dimensions. As will be described in the next section, we also develop relevant, but somewhat different, parameters based on similitude considerations.

Zook [17] lists approximately seventeen penetration equations along with the applicable target materials, projectile materials, projectile shapes, and striking velocities. The majority of this effort focuses on short $L/D$ projectiles such as fragments. Zook also reports the Thor coefficients for a number of target materials.

This review of the literature is not exhaustive, but it can be seen that penetration modeling has run the gamut of simple curve fits, more complex curve fits, and analytic models that attempt to predict penetration and perforation based on geometric and material properties. However, virtually all models require some empirical fit to one or more parameters to replicate experimental data. One of the most advanced analytical penetration models has been developed by Walker and colleagues [18–21]. The Walker–Anderson model, based on a momentum balance, uses an assumed flow field based on observations noted in hydrocode simulations [10]. The extent of the flow field is a weak function of the penetration