



# A theoretical study of shock front propagation in the density graded cellular rods



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## ABSTRACT

The present theoretical study focuses on the shock front propagation in a density graded cellular rod. It aims at the determination of the shock front velocity evolution as well as the stress evolution during the impact. The density gradient leads necessarily to the property gradient, which is taken into account in this analysis. The locking strain in classical shock front theory is replaced by the notion of the locking density to adapt to the studied case. Analytical solutions of linear, quadratic and square root density profiles are obtained. They are compared to the FEM solution with a good agreement. This study reveals a possibility to reduce the maximum impact stress for the impacted structures (same impacting mass, same energy absorption) using a proper density gradient.

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## 1. Introduction

Cellular materials (e.g. honeycomb, stochastic foam, hollow sphere agglomerate and lattice truss core material) have attracted important interests in both fundamental researches and industrial applications due to their high specific strength and good energy absorption capabilities. Therefore, a great number of experimental, numerical and analytical studies on the behaviour of cellular materials under quasi-static and dynamic loading have been reported in the past decades, showing that (i) the behaviour of a cellular material can be derived from its base material (cell wall material) and the relative density with a reasonable accuracy [7]; (ii) their strain rate sensitivities come not only from the base material rate sensitivity but also from structural effects [12,29].

Among those researches, an interesting feature is the shock front formation within the cellular materials under high speed impact. The pioneer study in the domain was carried out by Ref. [21]. They observed a huge strength increase when the balsa wood specimens were shoot at a velocity up to 250 m/s against an instrumented Hopkinson bar. In order to explain the measured

increases which seemed to be proportional to the square of the impact velocity, they developed a rigid perfectly plastic locking (RPPL) shock model.

The basic assumption of this model is that there exists a moving shock front separating the specimen into two zones: one (ahead of shock front) is rigid perfectly plastic and the other (behind the front) is compacted to a constant locking strain  $\epsilon_d$ .

The conservation laws lead to the following estimate of the stress behind the shock front  $\sigma_d$ , called afterward dynamic crushing stress:

$$\sigma_d = \sigma_p + \frac{\rho_0 V^2}{\epsilon_d} \quad (1)$$

This one-dimensional shock theory framework was successfully applied by a number of previous works to various metallic foams [9,20,25,26]. Experimental observations of those two distinct zones with high speed camera [6,18] gave a physical proof.

Improvements of such a theory were also reported. Ref. [15,16] proposed to take into account the initial elasticity with an elastic perfectly plastic rigid model (EPPR). Ref. [19] used a power law shock model to reduce the effect of a constant locking strain. Ref. [30] proposed another extension of RPPL model using a linearly hardening plastic locking model (R-LHP-L).

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Recently, the idea of functionally graded cellular materials was spread out. Ref. [8] investigated the hollow particles filled by functionally graded syntactic foam under quasi-static loading, in which the gradient was achieved by agglomerating different densities of hollow particles. Ref. [4] extensively studied the influence in energy absorption capability of foam with five different density gradient profiles under low speed impact loading and concluded that functionally graded foams were better in energy absorption than the uniform foam. Honeycomb-like structures with gradient were also theoretically and numerically investigated [1,2]. Ref. [28] investigated the influence of the density gradient profile on the mechanical properties of the graded polymeric hollow sphere agglomerates with Hopkinson bars techniques and direct impact tests. The experimental results showed that the gradient profile played an important role in both energy absorption and the transmitted force. Ref. [27] investigate the energy absorption capacity of density graded Voronoi honeycomb with a numerical method. Another kind of gradient, varying cross-section, can also be found [24].

With a property gradient, a very interesting question is the influence of the gradient on the shock front formation/propagation. Indeed [23], presented an analytical study of the shock front propagation in property graded cellular rods. They proposed that the plastic plateau stress of the cellular material varied with its position within the cellular rod while the initial relative density remained the constant. This assumption permits to apply easily the framework of RPPL model proposed by Ref. [21] with a constant locking strain. Results showed a significant influence of property gradient.

However, from the theoretical analysis provided by Ref. [7]; the quasi-static yield stress  $\sigma_p$  should be expressed as

$$\sigma_p = B \left( \frac{\rho_0}{\rho_s} \right)^{3/2} \sigma_{ys} \quad (2)$$

where  $\rho_0$  and  $\rho_s$  are respectively the density of the foam and that of base material,  $\sigma_{ys}$  denotes the yielding stress of the base material and  $B$  a constant determined by experimental data.

Thus, the gradient introduced by Ref. [23] was issued from an assumption where gradient was achieved by the yield stress variation of the base material  $\sigma_{ys}$  without any density change. However, in the industrial manufacture, the property gradient is generally obtained with the density gradients [8,11,28].

This paper tends to study the shock front formation/propagation in a cellular rod with property gradient due to relative density gradient. The main difficulty lies in the fact that the locking strain can not be constant so that the basic framework RPPL model in the previous literature can not be applied anymore. The basic assumption of this work is that the locking will take place at a given density. Indeed, the locking strain for cellular structures made of same base material and similar morphology varies with its initial porosities. However, at this locking point, the density of all those cellular material (same base material and morphology) is more or less the same. For example, if densification is idealised as a no porosity state, cellular structure made of same base materials will densify at the same base material density.

This assumption leads to an extension of the shock front theory to the case of the density gradient cellular rods. The theoretical analysis is presented and analytical solutions are found for linear, quadratic and square root gradient profiles. Finally, the effect of density gradient on the shock front formation/propagation in terms of resulting shock stress as well as energy absorption capacity are discussed in the cases of cellular rod with and without a backing mass.

## 2. Shock front in a cellular rod with density gradient

### 2.1. From locking strain to locking density

As mentioned in the introduction, the theory of shock formation in cellular materials is built with the notion of locking strain where cellular material begins to densify. This locking strain varies with the initial density of the cellular material and it is difficult to apply this notion for a cellular rod with a density gradient. However, this locking strain denotes the state where nearly all the initial voids have been compressed. It means also that at this state the density of the compressed cellular materials should have a given density (or given ratio between its density and the density of base materials). This idea leads to the notion of the locking density which is the starting point of our analysis.

The introduction of locking density is based on the theoretical prediction of densification strain:

$$\varepsilon_d = 1 - D \frac{\rho_0}{\rho_s} \quad (3)$$

This is a first order simplification, more accurate prediction should include an item of the third order of the relative density ( $\rho_0/\rho_s$ ) [3]. Actually, the first order form is widely used in literatures for simplicity. Ref. [7] suggested a value of 1.4 for the parameter  $D$  while [13] deduced the value of  $D (=4/3)$  by a theoretical analysis of honeycomb crush and this was also validated by FEM simulation. Based on these works, we can transform the Eq. (3) as

$$\varepsilon_d = 1 - \frac{\rho_0}{\rho_s/D} \quad (4)$$

Thus, locking density ( $\rho_d$ ) is introduced as following:

$$\rho_d = \frac{\rho_s}{D} \quad (5)$$

Therefore, the locking strain reads as Eq. (6).

$$\varepsilon_d = 1 - \frac{\rho_0}{\rho_d} \quad (6)$$

where  $\rho_0$  and  $\rho_d$  are the densities ahead of and behind shock front.

In order to verify this assumption, the ratio between the density at the locking strain and the base material density is calculated for the random cellular structures of various initial density (polymeric/aluminum foam, hollow sphere agglomerates, etc) available in open literature. All the experimental data leads to a more or less constant value. It manifests therefore that the locking density is rather independent to the initial density of cellular materials. Such an experimental observation is not exhaustive and it is only valid for listed materials and in the aforementioned range of impact velocity.

It was reported by the previous works of [25] and [31] that the locking strain is inevitably dependent on the impact velocity. Here, the data collected contains experimental results under quasi-static as well as moderate impact loading (<100 m/s). The case of extremely high speed plate impact testing (around km/s) are not considered in this work.

It is also emphasized that our study is limited in the case that the density gradient is rather small and the highest relative density of graded cellular bar is also small. With those limitations, the cell morphology variation and other induced structure effect due to density gradient can be neglected.

Therefore, in this studied density graded cellular rod, the locking density  $\rho_d$  is considered as a constant. However, the locking strain varies with the initial density of cellular materials, which depends on the current spatial coordinate.

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