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Accurate finite element simulation and experimental study of elastic wave propagation in a long cylinder under impact loading

A.V. Idesman [a,*], S.P. Mates [b]

[a] Department of Mechanical Engineering, Texas Tech University, Lubbock, TX 79409-1021, USA
[b] Metallurgy Division, MSEL, National Institute of Standards and Technology, 100 Bureau Drive Stop 8553, Gaithersburg, MA 20899, USA

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ABSTRACT

For the first time, detailed accurate numerical solutions for elastic wave propagation in a long axisymmetric elastic bar under impact loading are obtained using the new finite element technique proposed in our recent papers. In contrast to known numerical techniques, the new numerical approach quantifies and removes spurious high-frequency oscillations which may invalidate numerical results in impact loading simulations. The comparison of the accurate experimental results for the impact of striker and incident bars with the corresponding accurate numerical results allows us to explain some details of elastic wave propagation in long bars. For example, due to the absence of very high frequencies in the obtained experimental results, the mathematical formulation of the problem should include physical damping for the corresponding range of high frequencies. This range can be defined by the filtering stage of the new approach in terms of the number of finite elements along the radial direction of the bar. By the variation of this number we can fit the experimental curves with the numerical results obtained by the new numerical technique. However, for the accurate numerical solution of the impact problem with zero physical damping, the number of elements in the radial direction should be large. By the comparison of the numerical and experimental data, we can accurately determine the longitudinal wave velocity from experiments. The accurate numerical solutions also allow the analysis of the uniformity of the different strain and velocity components across the radius at different distances from the impact face. The validity of some assumptions used in the 1-D theory for wave propagation in long bars is also checked by the use of the accurate numerical solution. We have also shown that at the elastic impact the known dispersion-correction technique used for the description of the shape of the wave pulse at different locations along the axisymmetric bar is inaccurate for the prediction of pulses close to the impact face.

1. Introduction

The current study is motivated by the analysis of velocity, strain and stress distributions in the split Hopkinson pressure bar (SHPB). The SHPB is a very popular experimental technique for the study of mechanical properties of different materials at high rates of loading. This technique is based on the theory of wave propagation in elastic long bars. The 1-D theory yields a relatively simple analytical description of wave propagation, however, it is very inaccurate for high frequency pulses generated by impact loading. The axisymmetric and 3-D approaches based on integral transform techniques (e.g., see Refs. [1–7]) cannot yield closed-form analytical solutions for elastic wave propagation in a uniform cylinder (except in some asymptotic approximations); e.g., see a short description of such approaches in Ref. [8]. The semi-analytical technique used in Ref. [9] also requires the numerical solution of a system of equations. An approximation of the analytical solution for wave propagation in a cylinder is reported in Ref. [10] and is based on the Laplace transform and the numerical approximation of improper integrals. However, the solution in Ref. [10] contains spurious oscillations due to Gibbs phenomena (see curve 1 in Fig. 4) and is difficult to obtain with arbitrary initial and boundary conditions even for a simple geometry.

Relatively simple analytical techniques based on the dispersion correction (e.g., see Refs. [11–20] and others) are popular for the correction of the shape of wave pulses obtained in experiments in the SHPB. These techniques are based on the dispersion relation between the pulse frequency and the wave propagation rate in an

* Corresponding author.
E-mail address: alexander.idesman@coe.ttu.edu (A.V. Idesman).

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infinite cylinder; e.g., see Refs. [21,22]. However, as was mentioned in Ref. [23], there is no rigorous justification of the validity of the dispersion-correction method due to the application of the results for an infinite bar to the transient elastodynamics problem for a finite bar. We have not also seen in the literature that these dispersion correction techniques have been verified by the comparison with accurate numerical solutions at impact loading or at loading by high-frequency pulses.

A very general approach to the solution of wave propagation in a finite bar is based on the application of known numerical methods for elastodynamics; see Refs. [23–28] and others. Especially, the finite element method is very popular due to the simplicity of its application by the use of commercial finite element codes; e.g., see Refs. [23,25,26,28–33] and others. However, for high-frequency pulses, existing numerical approaches also do not allow an accurate solution of wave propagation problems due to large spurious high-frequency oscillations. For example, Fig. 1(a) shows the distribution of the axial velocity along the axis of revolution for the finite element solution at impact loading of an elastic cylinder with zero physical damping (see Section 3 for a detailed description of the problem). These results are obtained with linear finite elements with increasing mesh refinement and the implicit time integration by the trapezoidal rule with very small time increments. As can be seen from Fig. 1(a), the numerical results diverge with increasing mesh refinement and very small time increments and it is not clear what the solution is. We should mention that accurate solutions of elastodynamics problems are an issue with existing numerical methods based on the semi-discrete equations. Let us analyze this issue in greater detail. The application of finite elements in space (or other space discretization methods such as the boundary elements, spectral elements, the finite difference method and others) to linear elastodynamics problems leads to a system of ordinary differential equations in time

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{R}. \tag{1}$$

Here $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ are the mass, damping, and stiffness matrices, respectively, $\mathbf{U}$ is the vector of the nodal displacement, $\mathbf{R}$ is the vector of the nodal load. For wave propagation problems with zero damping ($\mathbf{C} = 0$), even the exact time integration of Eq. (1) may yield very inaccurate solutions of the original (before the space discretization) system of elastodynamics equations due to large spurious high-frequency oscillations (e.g., due to very small time increments used, the error in time can be neglected and the results in Fig. 1(a) are very close to the results with the exact time integration of Eq. (1)). Spurious high-frequency oscillations are related to the difference between numerical and exact solutions (i.e., to the oscillatory part of the numerical error), appear due to the application of the space-discretization method and are also affected by the size of time increments; e.g., see Ref. [34] for more details. Known tools for the suppression of spurious high-frequency oscillations in existing approaches are based on the introduction of artificial viscosity or numerical dissipation for a time integration method at each time increment. However, except our recent papers, we have not seen in the literature the quantifications of the frequency range of spurious oscillations as well as the amount of numerical dissipation or artificial viscosity. For example, in research and commercial finite-element codes the selection of the amount of numerical dissipation or artificial viscosity is user-defined and subjective because the accurate calibration of spurious oscillations is lacking. Therefore, in a popular textbook on the finite elements by Ref. [35], we can find the following statement: 'In no way can today's computer program for wave propagation and impact be treated as 'black boxes'. A minimum 6 months to 2 years of experience is needed to be able to use such programs successfully'. Another issue with the introduction of numerical dissipation or artificial viscosity at each time increment is the loss of accuracy at low modes at long-term integration; e.g., see Ref. [34]. This is very critical when we need a solution for wave propagation problems in long bars that requires a large number of time increments.

Recently we have developed a new accurate numerical approach for wave propagation problems (see Refs. [34,36–41] and the Appendix). This technique is based on the two-stage time integration approach with basic computations and the filtering stage, on the quantification of the range of spurious oscillations in finite element solutions and their filtering at the filtering stage (there is no necessity to filter spurious oscillations at each time increment), and on the finite elements with reduced numerical dispersion. With the new approach, there is no need for the assumptions related to the user-defined amount of artificial viscosity or numerical dissipation used in existing numerical approaches. For example, our results show that this amount should depend on the order of finite elements, the size of finite elements, the observation time, the numerical dispersion and other factors. The new approach yields numerical results converging to exact solutions with increasing mesh refinement; e.g., see the results in Fig. 1(b) after the filtering stage. With the new approach, finite elements codes can be used as 'black boxes' for wave propagation problems.

In this paper, we first shortly describe the experimental technique for the measurements of wave propagation in a long bar at impact loading (see Section 2). Accurate numerical simulations of wave propagation at impact loading based on the new numerical approach are presented in Section 3.1 with a detailed analysis of the obtained results. The comparison of the numerical and experimental results is given in Section 3.2. This comparison also leads to possible explanations of experimental results related to the frequency content of a pulse produced by an impact.

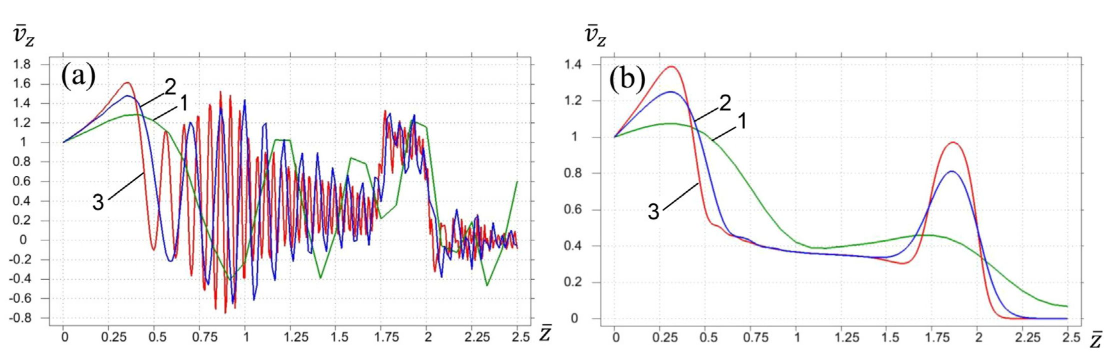

Fig. 1. The distribution of the dimensionless axial velocity $\bar{v}_z$ along the axis of revolution at dimensionless time $\overline{T} = c_0 t/R = 1.7241379308$ without (a) and with (b) the filtering stage. Curves 1, 2 and 3 correspond to the numerical solutions obtained on uniform meshes with 12, 60 and 150 finite elements in the radial direction, respectively.