International Journal of Machine Tools and Manufacture 132 (2018) 17–35

Contents lists available at ScienceDirect

International Journal of Machine Tools and Manufacture

journal homepage: www.elsevier.com/locate/ijmactool

Characterising the effects of shape on tool path motion

Luke Chanda, Robert J. Cripps*

Geometric Modelling Group, Department of Mechanical Engineering, University of Birmingham, Edgbaston, Birmingham, B15 2TT, United Kingdom

ARTICLE INFO

Keywords:
Tool path
Shape
Kinematics

ABSTRACT

This paper presents a methodology for *a priori* shape characterisation of tool path motion. Many current methods to describing tool path motion require explicit knowledge of the motion control algorithms implemented on a specific machine. Either a method proposes novel algorithms or requires knowledge of the algorithms currently implemented in a given machine's controller (e.g. minimum jerk, harmonic jerk and minimum jounce). This paper provides a method, that may be applied on any machine, to characterise motion in terms of a tool path's intrinsic shape properties. The characterisation identifies the achievable set of kinematics for a tool path of a given shape without the need for physical machining and a knowledge of the motion control algorithms. The characterisation may be employed in a pre-processing manner to inform the selection of NC file tool path motions. This can therefore help to reduce the material and energy resources being consumed during iterative machining trials and so improve the efficiency and productivity of the manufacturing process.

1. Introduction

To manufacture computer-aided design (CAD) models, computer-aided manufacturing (CAM) software can produce commands for computer numerically controlled (CNC) machines. Integral parts of these commands are descriptions of desired motions of the cutting tool relative to the workpiece. Such descriptions are commonly referred to as tool paths. In general, tool paths are discretised and presented to a machine's controller as a locus of poses. A single pose defines a tool's position and orientation. A tool path can also be considered as a locus of positions when a desired motion does not require changes in orientation.

A controller attempts to interpolate discretised tool paths by co-ordinating motion of independent translational and/or rotational axes. Linear and circular interpolation has traditionally been employed [1]. Piecewise-impulse and constant curvature profiles of such interpolated tool paths, can severely impede realisation of optimal kinematics and quality of the resulting machined component [2,3].

Consider a tangent discontinuous tool path composed of linear segments. The tangent vector at the junction of consecutive segments is not unique. A singularity in the tool paths curvature function exists at such a point. This corresponds to instantaneous change in direction, which is not possible in practice. To follow the path exactly, the cutting tool must come to rest at the junction. This intermittent motion requires changes in acceleration. The rate of change of acceleration, with respect to time, is defined as jerk [4]. The jerk experienced in such a motion can change resultant forces on the cutting tool, resulting in deflection marks on the surface of the machined component [5]. Also, fluctuations in feed rate, acceleration and jerk increase numerical control cycle time and in turn reduce productivity [6].

To reduce fluctuations in kinematic properties of tool path motion, CNC controllers can permit the actual path to deviate from tangent discontinuous junctions by a given tolerance [6]. By accelerating drives that will be active in the next segment and decelerating drives that are currently moving, the cutting tool is able to bypass a junction with a feed rate greater than zero. The greater the commanded feed rate, the greater deviation required to ensure kinematic limits of a given machine are not exceeded. The precise nature of the deviating motion may not be known to the user.

Due to machine manufacturers having ownership to the motion control algorithms implemented in controllers, their specifics are indeed not generally accessible to the engineers using the CNC machines [7]. It is for this reason that the effects of controller regulation on tool path motions are generally identified after numerous machining trials. To initially specify tool path motion parameters, such as feed rate and spindle speed, engineers therefore rely upon the recommendations of cutting tool manufacturers. These suggested values are however often based on assumptions of simple components with tool paths describing simple motions, predominately linear. Paths with varying curvature profiles, referred to in this paper as free-form paths, place a greater burden on control algorithms to generate the resulting motions.

Kinematics imposed by the shape of a free-form path, may exceed the capabilities of a given machine. For example, to maintain a specified feed rate, a machine must provide at each point, an acceleration

* Corresponding author.
E-mail address: r.cripps@bham.ac.uk (R.J. Cripps).

https://doi.org/10.1016/j.ijmachtools.2018.04.005
Received 11 December 2017; Received in revised form 16 April 2018; Accepted 17 April 2018
Available online 25 April 2018
0890-6955/

proportional to curvature. If unachievable, a machine's controller must moderate axes motions to provide permissible kinematics. In general a reduced feed rate is observed.

For a given application, the autonomous regulation of kinematics by a controller may produce undesirable and unknown machining conditions. A commanded feed rate may have been specified to achieve particular conditions, for example specific material removal rates or surface finish. NC file tool path motions may therefore need to be *optimised* for the given application. In such instances machinist experience can be significant [8].

The optimisation methods employed may be iterative and informed by empirical evidence from machining trials. Such *a posteriori* attempts to obtain suitable tool path motions are often heuristic and time intensive. Prior knowledge of the effects of specified NC file tool path motions, on the actual machine motions, can inform their selection. This in turn may reduce the time and number of machining trials thereby increasing the efficiency of the machining process.

It should be noted that knowledge of a specific motion algorithm only enables tool path motion description for the machine upon which it is used. The main contribution of this paper is that it proposes a methodology that may be employed on any machine tool in order to obtain a suitable description of tool path motion without the need for physical machining or knowledge of the control algorithms implemented in the given machine's controller. The resulting description of tool path motion depends only upon the intrinsic shape properties of a desired tool path and the kinematic limits of the given machine. Both these parameters are of the few conditions that are indentifiable without undergoing the iterative machining trial procedure. The paper therefore provides *a priori* shape characterisation of tool path motion. The characterisation may be employed to inform the selection of machining parameters and thereby reduce the time and the number of machining trials.

Currently, the primary means of describing tool path motion prior to physical machining is to acquire knowledge of the motion algorithms implemented in the machine's controller. However, as stated above, the algorithms are often not generally accessible. This has not deterred academia from proposing their own novel algorithms.

In general the proposed algorithms moderate both the commanded feed rate and tool path shape specified in the NC file in order to adhere to the machine's kinematic limits [9]. An algorithm may first fit a *smoother* path that interpolates the discrete NC file tool path poses whilst still adhering to the desired positional tolerances and then schedule an appropriate feed rate profile for the path's traversal [9].

Consider again the tangent discontinuous linearly segmented tool path. In order to combat the impractical kinematic demands imposed by the path's shape, a control algorithm may replace the junction between consecutive linear segments with a circular arc [6]. The direction of the tangent vector no longer changes instantaneously, thus enabling a continuous feed rate profile. Despite the revised tool path improving the motion, in the sense that traversal no longer requires an infinite acceleration, constant feed rate motion still cannot be realised. Although the imposed infinite acceleration has been removed, the new tool path's piecewise constant curvature imposes infinite jerk at the beginning and end of the arc. Attempts to produce these instantaneous changes in acceleration can excite vibrations in the mechanical structure of a machine and in turn degrade the dynamic performance of the servomotors [10]. It has been shown that constraining the permissible jerk experienced by each axis can limit the oscillatory behaviour of a machine and thus produce *smoother* tool path motions [11]. Thus many motion control algorithms consider limited-jerk, minimum-jerk and harmonic-jerk movement laws [11–13]. Further by constraining the magnitude of the resultant jerk vector the cutting tool may follow the revised tool path exactly [12,13].

Non Uniform Rational Basis Spline (NURBS) tool paths can also be implemented directly on many modern controllers. Motion control algorithms may use NURBS tool paths to avoid the impractical kinematic demands of linear and circular segments [14–17]. For example, two quartic polynomial splines can be used to achieve continuous curvature cornering within user specified tolerances [14]. A single G^2 quintic Bézier can be used to ensure axis acceleration limits are adhered to [15]. Many other proposed algorithms use B-splines as they offer flexibility in locally changing the shape of a tool path [16,17]. However, the polynomial nature of NURBS means that tool paths can experience undulating oscillations in curvature which in turn impose fluctuations in the kinematic properties of tool path motion [7,18].

As stated above, a key disadvantage of the current approaches is that they are only applicable to the given machines upon which the algorithms are implemented. The following sections of this paper describe and discuss an approach that may be employed on any machine in order to obtain a suitable characterisation of tool path motion without the need for physical machining or knowledge of the machine's motion control algorithms. Section 2 presents a description of machine motion in terms of tool path shape. Section 3 considers effects of kinematic limits on resulting motion. A Hermle C600U machine tool is then investigated as an example. First, in section 4, its kinematic limits are established by considering circular motions. Next, in section 5, planar spirals are traversed to consider the Hermle's behaviour to free-form path traversal. Section 6 details effects of shape in light of research undertaken. Finally, section 7 draws conclusions from key findings and suggests an area for future research.

2. Shape properties of motion

2.1. Local frame

The path traversed by a specified point on a cutting tool during a given motion can be represented as a parametric space curve, $\mathbf{r}(u)$, where u is an arbitrary parameter [19]. Assuming the path is continuous and differentiable, analysis of motion can be simplified by expressing it as a vector valued, time, t, parameterised function, in three dimensional Euclidean space, $\mathbb{E}^3$. $\mathbf{r}(t) = \langle x(t), y(t), z(t)\rangle$, $0 \le t \le T$, $x(t)$, $y(t)$ and $z(t) \in \mathbb{R}$ and T is the total time for the motion.

Employing a local coordinate system called the Frenet frame may facilitate describing each point in terms of shape properties of a tool path [18]. Considering changes in frame orientation as time increases can provide insight into effects of shape on tool path motion. As time increases it is assumed that the distance travelled along the path, arc length, s, increases monotonically. By reparameterising the path to arc length, frame motion may be expressed in terms of the path's intrinsic shape properties. Also a reparameterisation does not change the shape of the tool path. It then follows that $\mathbf{r}(s) \equiv \mathbf{r}(t)$, assuming $s = s(t)$ is differentiable and the inverse $t = t(s)$ exists. By letting $s(t) = \int ||\dot{\mathbf{r}}(t)||dt$ be such a parameterisation,

$$\frac{d\hat{\mathbf{t}}(s)}{ds} = \kappa(s)\hat{\mathbf{n}}(s)\,, \tag{1}$$

$$\frac{d\hat{\mathbf{n}}(s)}{ds} = -\kappa(s)\hat{\mathbf{t}}(s) + \tau(s)\hat{\mathbf{b}}(s)\,, \tag{2}$$

and

$$\frac{d\hat{\mathbf{b}}(s)}{ds} = -\tau(s)\hat{\mathbf{n}}(s)\,. \tag{3}$$

In Eqs. (1)–(3), $\hat{\mathbf{t}}(s)$, $\hat{\mathbf{n}}(s)$ and $\hat{\mathbf{b}}(s)$ are defined as the unit tangent, principal normal and binormal vectors respectively and functions $\kappa(s)$ and $\tau(s)$ refer to shape properties curvature and torsion respectively [20]. Formally $\kappa(s)$ and $\tau(s)$ are defined by Eqs. (1) and (3) respectively. They can also be considered to describe frame motion and in turn any object that moves with the frame, for example a cutting tool. As a Frenet frame traverses a path, $\kappa(s)$ and $-\tau(s)$ describe the angular velocities of unit tangent and binormal vectors [18]. Further, curvature and torsion are independent of path parameterisation. They are