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A real-time look-ahead interpolation algorithm based on Akima curve fitting

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ABSTRACT

To process the complex geometric-shapes consisting of discrete data points, this paper presents a real-time look-ahead interpolation algorithm based on Akima curve fitting. The algorithm consists of two modules: pretreatment module and real-time interpolation module. The pretreatment module firstly adopts the bi-chord and tangent-chord criteria to identify those continuous Akima blocks and calculates the linking velocity between two adjacent blocks according to the required machining accuracy. Then the five-point Akima method is used to fit the continuous blocks into a Akima spline curve. In the real-time interpolation module, the look-ahead interpolation is executed to ensure that the processing error is limited in a satisfying range, and the Newton iteration method is employed to calculate the parameters when interpolating Akima spline blocks. Simulations and experimental results demonstrate that the Akima spline is more suitable when processing discrete data points. And the proposed look-ahead interpolation algorithm could reduce the feedrate fluctuation to satisfactory level and implement high-quality CNC processing.

1. Introduction

Conventional CNC machines only provide linear and circular interpolators, so most of complex geometric-shape parts designed with CAD/CAM systems have to be approximated with many small linear and circular segments. However, this method has certain problems: firstly, for machining complex part shapes, the size of the NC data file is very large, so that the DNC transfer load is increased; secondly, at the junctions of the line segments, acceleration discontinuity and excessive jerk would occur at high speed, so the system vibration is easily arising and the machining quality would be reduced [1,2]. To overcome the disadvantages of traditional CNC systems, some high-end commercial CNC systems have realized parametric interpolators. Take SIEMENS840D for example, its spline interpolator could fit the data points into an A(Akima) curve, a NURBS (Non-Uniform Rational B Spline) curve or a C (Cubic) curve [3], as shown in Fig. 1. However, these techniques have not been discussed openly. Among these spline curves, NURBS interpolation receives more attention. Yeh and Su [4] adopted the least-squares approach method to fit the discrete data points into the NURBS curve. Wang and Yau [5] applied the critical corner angle test and the bi-chord error test to verify which linearized segments could be regarded as continuous NURBS blocks. However, there are three difficult issues in employing real-time NURBS interpolation: (I) heavy computation load on recursive algorithms, (II) inevitable errors introduced by employing a truncated Taylor series [6] and (III) complex fitting conditions caused by the geometric characteristics of NURBS curve. Tsai et al. [7] developed a real-time look-ahead interpolation methodology with a Bezier spline-fitting technique. But this method needs to calculate and attain two additional points, so the matrix operations are introduced, which is very time-consuming. Zhao et al. [8] adopted the B-spline curve to blend the adjacent straight lines, but the mathematical calculation of B spline and the adaptive subdivision scheme used in the method were quite complex. Zhang et al. [9] applied the Cubic spline curve to interpolate the fitting path. Unlike parametric curves above, C spline curve passes exactly through all the data points. But the C spline curve is not local, one data points alteration could influence a large number of blocks. So it can only be used in cases where the interpolation points lie along an analytically calculated curve. The Akima spline curve could also pass exactly through all the data points, but the Akima spline curve is local, a change to an interpolation point affects only up to six adjacent points [10]. But the Akima curve just has the first-derivative continuity, only the first-order Taylor's expansion

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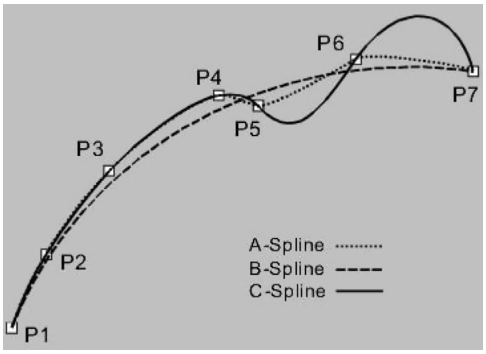

Fig. 1. Comparison of three spline types with identical data points of SIEMENS interpolator.

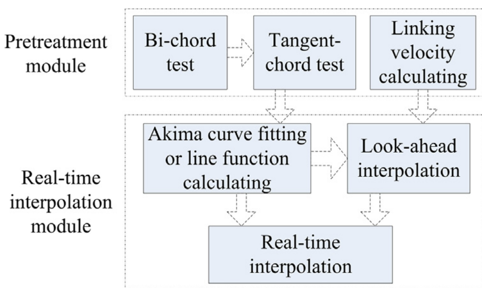

Fig. 2. The system architecture of the proposed algorithm.

can be used to calculate the spline parameters (the second-order Taylor's expansion could result to the discontinuity of theprocessing path), which may lead to great feedrate fluctuation.

In this paper, a real-time look-ahead interpolation algorithm based on Akima curve fitting is proposed. The algorithm consists of two modules: pretreatment module and real-time interpolation module, as shown in Fig. 2. Because of the constraints of processing precision, not all the linear G01 short blocks could be fitted into a smooth Akima curve. Hence, the pretreatment module firstly adopts the bi-chord error test and the tangent-chord error test to identify those continuous Akima blocks and calculates the linking velocity between adjacent micro-line or spline segments according to the required machining accuracy and corner angle. Then in the real-time interpolation module, the five-point Akima method is employed to generate the Akima curve of degree three, and the parametric function of micro-line is used to unify the process of line interpolation and spline interpolation. Before real-time interpolating, the look-ahead interpolation is carried out to guarantee the processing accuracy. Finally, during the real-time Akima interpolation phase, the Newton iteration method is employed to calculate interpolation parameters, which can dominate the feedrate fluctuation to an ideal level.

2. Introduction to Akima spline curve

2.1. Five-point Akima method

The five-point Akima method is based on a piecewise function composed of a set of polynomials, each of degree three [11]. For a set of data points, $P_0, P_1, P_2 \ldots P_{n-1}$, the corresponding parameters are $u_0, u_1, u_2 \ldots u_{n-1}$, and satisfy $u_0 < u_1 < u_2 \ldots u_{n-2} < u_{n-1}$. In the subinterval $[u_i, u_{i+1}]$ $(i=0,1\ldots n-2)$, the Akima curve function $R(u)$ is partly defined as

$$R(u) = A + B(u-u_i) + C(u-u_i)^2 + D(u-u_i)^3 \tag{1}$$

In order to calculate the coefficient vectors A, B, C and D of the polynomial, the first derivatives $R'(u_i)$ and $R'(u_{i+1})$ are applied. In Akima's method, $R'(u_i)$ is replaced by the approximated value $s(u_i)$, which is estimated by

$$s(u_i) = \frac{|U_{i+1} - U_i| U_{i-1} + |U_{i-1} - U_{i-2}| U_i}{|U_{i+1} - U_i| + |U_{i-1} - U_{i-2}|} \tag{2}$$

where U_i is the slope vector of the linear segment P_iP_{i+1}. If the data points are two-dimensional, and P_i's coordinate is (x_i, y_i), P_{i+1}'s coordinate is (x_{i+1}, y_{i+1}), then U_i can be determined by

$$U_{i_x} = \frac{x_{i+1} - x_i}{u_{i+1} - u_i}, \quad U_{i_y} = \frac{y_{i+1} - y_i}{u_{i+1} - u_i} \tag{3}$$

In the beginning and final of the curve, four more slope vectors need to be estimated according to the trend of the given points, which can be described as

$$\begin{cases} U_{-1} = 2U_0 - U_1 \\ U_{-2} = 2U_{-1} - U_0 \\ U_n = 2U_{n-1} - U_{n-2} \\ U_{n+1} = 2U_n - U_{n-1} \end{cases} \tag{4}$$

From Eq. (2), it is seen that $s(u_i)$ depends only on five data points $P_{i-2}, P_{i-1}, P_i, P_{i+1}, P_{i+2}$ and is independent of the interval widths. Finally, in the subinterval $[u_i, u_{i+1}]$, the coefficients of the Akima curve can be calculated by

$$\begin{cases} A = Cd_i \\ B = s(u_i) \\ C = \dfrac{3U_i - 2s(u_i) - s(u_{i+1})}{u_{i+1} - u_i} \\ D = \dfrac{-2U_i + s(u_i) + s(u_{i+1})}{(u_{i+1} - u_i)^2} \end{cases} \tag{5}$$

where Cd_i is the coordinate vector of point P_i.

2.2. Parameterization of the data points

The parameters of data points should reflect the nature of spline curve. The accumulative chord length method is now the most widely used and accepted method. But this method is most suitable only if the parametric curve is a straight line. So the centripetal parameterization method proposed by Lee [12] is employed to calculate the parameters of the discrete data points. The method uses the square root of accumulative chord length to calculate the node parameters. The equation is given as

$$\begin{cases} u_0 = 0 \\ u_i = u_{i-1} + \dfrac{\sqrt{|P_i - P_{i-1}|}}{\sum \sqrt{|P_i - P_{i-1}|}}, \quad i = 1, 2 \ldots n-1 \end{cases} \tag{6}$$

3. Pretreatment

To realize Akima curve interpolation, the first task of the pretreatment module is to determine whether the data points could be used in spline fitting. So the bi-chord test is taken into consideration. According to [6], if neighboring blocks pass the bi-chord test, these blocks could be regarded as continuous short blocks. But the fitting curve may not satisfy the processing accuracy. To overcome this problem, the tangent-chord test is adopted to determine whether those continuous blocks could be fitted into the Akima curve.