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Symmetric snap-through and equal potential energy load of non-uniform shallow arch under a concentrated load considering imperfection effect

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ABSTRACT

This paper deals with analytical investigation of non-uniform circular shallow arch under a concentrated load considering imperfection effect under pinned-pinned boundary conditions. After characterizing the non-uniformity by three piecewise constant stiffness segments, governing equilibrium equations including imperfection effect are derived via least potential energy principle and criteria for possible occurrence of symmetric snap-through are explicitly presented by using L'Hospital's rule after identifying two modified slenderness parameters. The existence of an equal potential energy load is rigorously shown in a straightforward manner and its parametric dependence on imperfection and non-uniformity is discussed. The relationship between equal potential energy load and buckle propagation load is studied by 3D FEA of buckle propagation of corresponding non-uniform shallow panel. Moreover, two limiting cases including rigid end case and rigid center case are analyzed by employing augmented potential energy with Lagrangian multipliers. An asymptotical solution shows that a finite modified slenderness (independent of imperfection) is sufficient to ensure the occurrence of symmetric snap-through when dimension of center segment is approaching to zero in rigid end case and this conclusion holds when there is some rotational restraint at ends. For rigid center case, the closed-form criterion considering imperfection for symmetric snap-through occurrence is presented. Equal potential energy load for two limiting cases is derived and analyzed in detail. This paper is expected to enhance the understanding of stability of imperfect non-uniform shallow arch.

1. Introduction

Shallow archesQ serve commonly as load carrying structures such as bridges, house roofs. Nowadays, the symmetric snap-through buckling phenomenon of shallow arch has not just been regarded as failure. But in some cases, symmetric snap-through buckling of shallow arch has been utilized to design some smart micro-structures such as micro-pump, micro-valve. The buckling of shallow arch is distinctly different from the buckling of rings, since the pre-buckled state of shallow arch is highly non-linear [1] and the pre-buckling of rings could be assumed to be linear [2]. The overestimation of classic bifurcation solution considering linear pre-buckling behavior for shallow arch is shown by Pi et al. [1]. In some cases, after symmetric snap-through, the shallow arch reaches a distant equilibrium state with axial force being tension instead of compression. Schreyer and Masur [3] seems the first to conduct exact analysis of shallow arch analysis under uniform radial pressure or a concentrated load. Gjelsvik and Bodner [4] conducted experimental studies and formulated an approximate solution for shallow arch under a concentrated load taking geometric imperfection effect into account.

The recent development of shallow arch buckling research is notably focused on the various boundary conditions, load types and materials.

For various boundary conditions, Pi et al. [5–7] extensively discussed the effect of rotational restraint at ends and analyzed the cases of both equal and unequal rotational stiffness. An interesting finding is that for unequal rotational stiffness, the shallow arch doesn't buckle by bifurcation so that the only failure mode is symmetric snap-through. This result is in direct contrast with the case of equal rotational stiffness where both bifurcation and symmetric snap-through are possible. The analytical and experimental study of the horizontally supported steel shallow arch has been carried out recently by Han et al. [8] and Lu et al. [9]. Another interesting research is about shallow arch rotationally restrained at ends by two rotation springs whose stiffness is load dependent. In this respect, Plaut [10] investigated the buckling of shallow sinusoidal arches with rotational supports that stiffen when compressed. Similarly recently Cai and Feng [11,12] investigated the buckling of shallow parabolic arch with rotational supports that stiffen when compressed.

For various load types and various materials, Pi and Bradford [13] studied the non-linear in-plane elastic buckling of shallow circular arches under both uniform radial pressure and thermal loading. Bateni and Eslami [14] investigated the effect of temperature gradient on the

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Nomenclature

R	radius of the circular shallow arch
A_1,A_2	cross-section areas for center segment and end segments respectively
I_1,I_2	moments of inertia for cross-sections of center segment and end segments respectively
$D_{\max}$	magnitude of initial imperfection magnitude
D_m	dimensionless imperfection parameter $D_m = D_{\max}/{\theta_3}^2$
$\hat{D}$	initial imperfection displacement
D	dimensionless initial imperfection displacement $D = \hat{D}/R$
E	Young's modulus
υ	Poisson's ratio for 3D panel analysis (equals to 0.3 in this paper)
θ_1,θ_2	angular sizes of center segment and end segments(see Fig. 1)
θ_3	total angle of arch $\theta_3 = \theta_1 + \theta_2$
s	angular coordinate of shallow arch(see Fig. 1)
β_1,β_2,α_1,α_2	non-uniformity parameters defined as $\beta_1 = EI_2/EI_1$; $\beta_2 = 1/\beta_1$; $\alpha_1 = EA_2/EA_1$; $\alpha_2 = 1/\alpha_1$
λ	$\lambda = \theta_1/\theta_3$
γ_1,γ_2	slenderness parameters by ${\gamma_1}^2 = EI_1/EA_1R^2$, ${\gamma_2}^2 = EI_2/EA_2R^2$
λ_{s1},λ_{s2}	modified slenderness parameters as $\lambda_{s1} = {\theta_3}^2/\gamma_1$,$\lambda_{s2} = {\theta_3}^2/\gamma_2$
Q	magnitude of central concentrated force
N	membrane force of center segment and end segments
μ_1,μ_2	$\mu_1 = \sqrt{NR^2/EI_1}$ and $\mu_2 = \sqrt{NR^2/EI_2}$
μ_{1T},μ_{2T}	$\mu_{1T} = \sqrt{-NR^2/EI_1}$, $\mu_{2T} = \sqrt{-NR^2/EI_2}$
$\hat{\mu}_1$	$\hat{\mu}_1 = \mu_{1T}$ for axial tension solution, $\hat{\mu}_1 = \mu_1$ for axial compression solution
$\bar{Q}$	normalized load as $\bar{Q} = QR^2\theta_3/EI_1$
$\tilde{Q}$	normalized load as $\tilde{Q} = QR^2\theta_3/EI_2$
$\overset{\frown}{Q}$	$\overset{\frown}{Q} = \tilde{Q}$ in stiffer center case or rigid center case and $\overset{\frown}{Q} = \bar{Q}$ in stiffer end case or rigid end case
$\overset{\frown}{\lambda}$	$\overset{\frown}{\lambda} = 1 - \lambda$ in stiffer center case or rigid center case and $\overset{\frown}{\lambda} = \lambda$ in stiffer end case or rigid end case
$\overset{\frown}{\alpha}$	$\overset{\frown}{\alpha} = \alpha_1$ in stiffer center case or rigid center case and $\overset{\frown}{\alpha} = \alpha_2$ in stiffer end case or rigid end case
$\hat{w}_1$, $\hat{w}_2$	tangential displacements of center segment and end segments respectively
$\hat{v}_1$, $\hat{v}_2$	radial displacement of center segment and end segments respectively
w_1,w_2,v_1,v_2	normalized displacements $w_1 = \hat{w}_1/R$, $w_2 = \hat{w}_2/R$, $v_1 = \hat{v}_1/R$, $v_2 = \hat{v}_2/R$
v_c	radial displacements of crown point $v_c = v_1\|_0$
$\bar{x}$, $\bar{y}$	imbedded coordinates of cross-section(see Fig. 1)
ε_{1m},ε_{2m}	membrane strains of center segment and end segments respectively
ε_{1b},ε_{2b}	bending strains of center segment and end segments respectively
ε_1,ε_2	total circumferential strains
Π	total potential energy of the whole system
Π^*	augmented potential energy
λ_1,λ_2	Lagrangian multipliers
x^*, $\hat{x}$	least positive roots of some transcendental equations
K	rotation stiffness of rotational spring at ends(see Section 4.2.2.2)
k	normalized rotation stiffness defined as $k = \theta_3KR/EI_1$
$\delta()$,$()'$	Variation of ( ) and derivative of ( ) with respect to s
t_1,t_2	thickness of center segment and end segments respectively with rectangular cross-section assumed
h	width of cross-section with rectangular cross-section assumed
Z	axial coordinate of 3D shallow panel
L	length of 3D shallow panel
u_y	displacement of leading generator of shallow panel in Y direction
U	generalized displacement defined as integral $\int_0^L (-u_y)dZ$
H	non-dimensional axial compression $H = \sqrt{\lambda}\mu_1\theta_3$ for asymptotic analysis
$\hat{Q}$	non-dimensional load defined as $\hat{Q} = \lambda\bar{Q}/(1-2D_m)$ for asymptotic analysis
Load curves Meaning	$\bar{Q} - \mu_1\theta_3$ curves or $\tilde{Q} - \mu_2\theta_3$ curves

mechanical buckling resistance of shallow arches made of functional gradient material FGM. Furthermore FGM shallow arch's collapse under a central concentrated load and uniform radial pressure was studied by Bateni and Eslami [15,16]. The shallow arch under electric loads has been analyzed for microelectromechanical applications by Das and Batra [17]. Under electric load, there is an additional instability named pull-in instability. New analytical formulations were derived by Kiss and Szeidl [18–20] to investigate the buckling of heterogeneous shallow arch under a concentrated load for fixed-fixed boundary conditions, rotationally restraint boundary condition. Instead of analyzing the load control case, Plaut [21, 22] analyzed the shallow arch under unilateral displacement control analytically by treating the arch as inextensible elastica and calculated the critical displacements for symmetric snap-through. This study emphasizes on the effect of load position.

Despite the numerous study of instability of shallow arch, only a few researches are about the non-uniform shallow arch. For example, Tsiatas and Babouskos [23] developed an interesting "analog equation method" to study non-linear instability of non-uniform shallow arches for fixed-fixed and pinned-pinned boundary conditions. It is worthy to mention that Yan et al. [2,24], Xue and Fatt [25] , Fatt [26], Ye et al. [27] and Shen et al. [28] have investigated the buckling of cylindrical shells under external pressure and characterized the non-uniformity by piecewise constant thickness distribution. Adopting this distribution, recently Yan et al. [29, 30] systematically investigated the static instability of non-uniform shallow arch under a concentrated load or uniform radial pressure for pinned-pinned boundary conditions and fixed-fixed boundary conditions.

In these previous studies, the non-uniform shallow arch has been shown to behave notably differently from the uniform shallow arch. Especially when some parts of arch are rigid, there exists an interesting asymptotical structure when the dimension of center segment is approaching to zero. However, in previous research for non-uniform shallow arch by the authors [29,30,38], the imperfection has not been fully considered. A related investigation for uniform shallow arch by Zhuangpeng [31] and previous brief study by Yan et al. [38] both show that the