International Journal of Mechanical Sciences 145 (2018) 299–317

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Generalized actual inverse kinematic model for compensating geometric errors in five-axis machine tools

Yang Liu [a,b], Min Wan [a,b,*], Wan-Jing Xing [a,b], Qun-Bao Xiao [a,b], Wei-Hong Zhang [a,b,*]

[a] *School of Mechanical Engineering, Northwestern Polytechnical University, Xi'an, Shaanxi 710072, China*
[b] *State IJR Center of Aerospace Design and Additive Manufacturing, Northwestern Polytechnical University, Xian, Shaanxi 710072, China*

ARTICLE INFO

Keywords:
Five-axis machine tools
Geometric errors
Screw theory
Actual inverse kinematic model
Compensation

ABSTRACT

Geometric errors of five-axis machine tools, i.e. position independent and position dependent geometric errors (PIGEs and PDGEs), should be compensated in order to improve the machining precision of workpieces. To achieve this purpose, this paper proposes a generalized actual inverse kinematic model (IKM), which provides explicit solution for the compensated motion commands and can be directly applied to five-axis machine tools with arbitrary configurations, especially for those with non-orthogonal rotary axes. The typical characteristic of this model lies in three aspects. First, it is the first effort to compensate both PIGEs and PDGEs in a generalized way. Second, it provides an explicit solution for changing the motion commands of the machine tool's axes so that the geometric errors can be compensated. Third, as the configuration of machine tool changes, the model can be directly used without additional theoretical derivations, which is usually required by existing methods. Rotation-constrained equation is newly formulated and its solution is derived in detail to obtain the compensated motion commands of rotary axes. By introducing the incremental motion commands of translational axes, the solution to the actual IKM does not explicitly include the geometric error items and the model's formulations are greatly simplified. Simulation results verify the effectiveness, feasibility and universality of the proposed method. Experimental results confirm that the machined workpiece has a remarkable precision improvement by using the proposed compensation method.

1. Introduction

Five-axis machine tools have been widely used to machine parts with free-form surfaces such as dies, molds and aerospace parts. The increasing demands for improving the precision of these parts urgently require to improve the machine tools' machining accuracy. During the machining process, the factors influencing the machining accuracy include geometric errors [1,2], thermal errors [3,4], servo errors [5,6], force or wear-induced static errors [7–9] and vibration-induced errors [10–12]. Among them, geometric errors of machine tools are one of the biggest sources of inaccuracy [13], which draw a lot of attention recent years. According to the definitions in Ref. [14], geometric errors of machine tools can be classified as position independent geometric errors (PIGEs) and position dependent geometric errors (PDGEs). PIGEs are caused by the imperfect assembling of the machine tool components and are approximately treated as constants, while PDGEs are induced by the imperfection of machine tool components and vary from position to position in the workspace. It has always been a key issue to identify the geometric errors and further compensate the effects of them. Many methods, which were established for identifying the geometric errors of five-axis machine tools, can achieve remarkable identification precision for both translational and rotary axes with the help of some instruments such as laser interferometer [15,16], laser tracker [17,18], double ball bar [19,20], Doppler laser instrument [21], touch trigger probe [22] and 3D probe-ball [23].

Once the geometric errors are identified, they should be compensated for the aim of enhancing machining accuracy. Currently, developing tool path modification method to compensate geometric errors is a hot research topic. The kernel idea of tool path modification is to modify the ideal motion commands, which are calculated by the ideal inverse kinematic model (IKM), as the compensated motion commands to ensure that the tool tip positions and tool orientations relative to the workpiece coordinate system (WCS) are the desired ones. The ideal expectation is that the compensated motion commands could be directly calculated through the actual IKM, which is the IKM under the influences of geometric errors, since it can both acquire high compensation precision and improve the calculation efficiency [24,25]. However, the existence of the two rotary axes makes the ideal forward kinematic model (FKM) of five-axis machine tools highly nonlinear, and thus, derivation

---

[*] Corresponding authors at: School of Mechanical Engineering, Northwestern Polytechnical University, Xi'an, Shaanxi 710072, China.
E-mail addresses: m.wan@nwpu.edu.cn (M. Wan), zhangwh@nwpu.edu.cn (W.-H. Zhang).

https://doi.org/10.1016/j.ijmecsci.2018.07.022
Received 14 February 2018; Received in revised form 30 June 2018; Accepted 18 July 2018

of the actual IKM involving the numerous geometric error items becomes rather difficult. Hence, lots of researchers obtained the compensated motion commands through establishing other methods, such as linearized methods, differential methods and iterative methods. Comments of these methods will be reviewed as follows.

Linearized methods approximate the non-linear relationship between the geometric errors and the compensated motion commands as a linear one. Lei and Hsu [26] assumed that the small variations of tool poses in the WCS have a linear relationship with the incremental motion commands in the machine coordinate system (MCS), based on which, the compensated motion commands were obtained. Tsutsumi et al. [27] built an approximate linear relationship between geometric errors and the compensated motion commands of translational axes, which was then used to establish a simplified compensating procedure. This procedure is able to obtain precise tool tip positions, however, the desired tool orientations can not be ensured since orientation errors were ignored. Actually, the desired tool orientations relative to the WCS should also be achieved to avoid undercut and overcut.

Differential methods aim at building the relationship between the tool pose errors and the incremental motion commands. The compensated motion commands are then obtained by adding the incremental motion commands to the ideal motion commands. Fu et al. [28] established the differential motion matrix (DMM) of each axis relative to the tool coordinate system (TCS) to construct Jacobian matrix, whose pseudo-inverse was further used to obtain the incremental motion commands. Bi et al. [29] and Huang et al. [30] established the relationship between the tool pose errors and the incremental motion commands by differentiating the ideal FKM, and then calculated the incremental motion commands by using the least square algorithm.

There are also some researches focusing on developing iterative methods. By assuming that there are no differences between the geometric errors corresponding to the ideal motion commands and those related to the compensated motion commands, Zhu et al. [19] presented an iterative procedure to obtain the compensated motion commands through solving the ideal IKM. Aguado et al. [17] proposed an iterative optimization method to compensate the geometric errors. They used laser tracker to measure the geometric errors, and then realized constructing the geometric error information in the whole workspace by using Chebyshev polynomials. Xiang and Altintas [31] improved Zhu's method [19] by introducing the differences between the geometric errors associated with the ideal motion commands and the compensated motion commands. Peng et al. [18] commented that solving scheme of the IKM will lower the efficiency of the iterative procedure for obtaining the compensated motion commands, and thus, they proposed a total differential algorithm to improve the iterative efficiency. They obtained the geometric errors of multi-axis CNC machine tool through using laser tracker to construct the centerlines of translational and rotary axes. Later, Zhou et al. [32] also improved the iterative efficiency by proposing a decoupled compensation method based on the topology relation between each axis in the kinematic chain of five-axis machine tools. Lately, Wu et al. [33] proposed an iterative method for five-axis machine tools with non-orthogonal rotary axes based on the relative motion constraint equation.

Besides linearized methods, differential methods and iterative methods, there also exist some beneficial attempts aiming to establish the actual IKM, which have the advantage of efficiently providing explicit solution for the compensated motion commands. Yang et al. [24] firstly proposed an actual IKM by including PIGEs through calculating the twist errors using screw theory. Ding et al. [25] developed an actual IKM using homogeneous transformation matrix (HTM) to obtain the analytical expressions of the compensated motion commands according to the invertibility of HTM and the rotation-invariant feature. It should be mentioned that the rotation-invariant feature is only suitable for five-axis machine tools with orthogonal rotary axes, whose three linear axes are orthogonal to each other, and the centerline of rotary axis is parallel to the linear axis.

Note that except the approach proposed by Yang et al. [24], all other geometric error compensation methods mentioned above are established based on the commonly used HTM products. These compensation methods are helpful for improving the machining accuracy, while none of them can provide uniform and explicit mathematical formulations of the compensated motion commands for machine tools with arbitrary configurations, and the model built for a specific five-axis machine tool is difficult to be directly adopted to machine tools with other configurations due to the following fact. The HTM products require to establish local coordinate system for each moving axis relative to the previous local coordinate system in the whole kinematic chain during the geometric error modeling procedure. This means that for machine tools with different configurations, different sets of local coordinate systems are required. As a result, complicated and lengthy mathematical formulations, which are used to calculate the compensated motion commands, are needed to be re-derived for each machine tool. This greatly suppresses the universality of these compensation methods. Especially, Sato [34] and Moriwaki [35] reported that there are 216 kinematically feasible configurations to achieve five-axis machine tools by changing the order of linear and rotary axes. Besides, various machine tool builders, such as Makino, Ingersol and Deckel Maho, develop five-axis machine tools with non-orthogonal rotary axes, whose rotary axes are in an inclined plane, to further improve their versatility and flexibility [36,37]. This fact further expands the amount of possible configurations of five-axis machine tools. In this situation, it is of great significance to develop a generalized compensation method for five-axis machine tools with arbitrary configurations. The invention of screw theory [38,39] provides a method to globally describe the rigid body motions. There is no need to assign local coordinate systems as required by the HTM products, and all vectors and points are defined in the one and only reference coordinate system. In this way, it is possible to develop generalized motion equations of mechanisms by using screw theory. Screw theory is firstly applied to the robotic field [40,41] and then introduced to study machine tools [42,43]. Yang et al. [24] are the pioneers in developing generalized compensation method using screw theory by including the effects of PIGEs. However, to the authors' best knowledge, there are no reports on generalized compensation method by simultaneously including the effects of both PIGEs and PDGEs.

This paper proposes a generalized actual IKM for compensating the geometric errors of five-axis machine tools by considering both PIGEs and PDGEs. It provides uniform and explicit actual inverse kinematic solution for five-axis machine tools with arbitrary configurations. Specially, it is suitable for five-axis machine tools with either orthogonal or non-orthogonal rotary axes without the need of additional theoretical derivations. Actual FKM is firstly formulated in Section 2 by integrating both PIGEs and PDGEs into the ideal one. Based on the basic principles of screw theory, the rotation-constrained equation is then defined from the obtained actual FKM. By solving this subproblem, the compensated motion commands of rotary axes are derived in detail. Subsequently, the compensated motion commands of rotary axes are substituted into the actual FKM to obtain the incremental motion commands of translational axes, which are further added to the ideal motion commands to obtain the compensated motion commands of translational axes. These contents are detailed in Section 3. Section 4 gives numerical and experimental verifications, followed by conclusions in Section 5.

2. Generalized actual forward kinematic model

The generalized actual forward kinematic model (FKM) is the base of the proposed generalized actual inverse kinematic model (IKM), which is firstly constructed as follows.

Three coordinate systems are defined, i.e. the reference coordinate system (RCS), the workpiece coordinate system (WCS) and the tool coordinate system (TCS), as can be seen from Fig. 1. The RCS ($O_rX_rY_rZ_r$) is attached to the machine tool base, and its origin is set at the intersection between the centerlines of the two rotary axes when all axes are in their