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Comparison of inverse identification strategies for constitutive mechanical models using full-field measurements

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A B S T R A C T

The calibration of phenomenological constitutive material models has been a constant need, because the parameters differ for each material and the ability of a model to mimic the real behaviour of a material is highly dependent on the quality of these parameters. Classically, the parameters of constitutive models are determined by standard tests under the assumption of homogeneous strain and stress fields in the zone of interest. However, in the last decade, Digital Image Correlation techniques and full-field measurements have enabled the development of new parameter identification strategies, such as the Finite Element Model Updating, the Constitutive Equation Gap Method, the Equilibrium Gap Method and the Virtual Fields Method. Although these new strategies have proven to be effective for linear and non-linear models, the implementation procedure for some of them is still a laborious task. The aim of this work is to give a detailed insight into the implementation aspects and validation of these methods. Detailed flowcharts of each strategy, focusing on the implementation aspects, are presented and their advantages and disadvantages are discussed. Moreover, these modern strategies are compared for the cases of homogeneous isotropic linear elasticity and isotropic plasticity with isotropic hardening. A simple numerical example is used to validate and compare the different strategies.

1. Introduction

With the innovation surge currently happening in industry, reliable and fast solutions for engineering problems are more important than ever. Numerical simulation has been a valuable tool for their resolution and is now well-established. However, it is essential for these tools to keep a continuous improvement of their predictive capabilities. One of the areas for potential improvement is mechanical modelling of materials and the respective calibration procedure. The quest for more accurate models has been particularly intense regarding the elasto-plastic behaviour of sheet metals. Indeed, many advanced and more complex mechanical models have been developed to accurately describe phenomena such as hardening and anisotropy. However, this increase in complexity usually means a tedious process of parameter calibration, due to long experimental campaigns. For example, the yield criterion Yld2000 [1] depends on 8 material parameters, which requires three uniaxial yield stresses and three uniaxial anisotropy coefficients, the biaxial yield stress and anisotropy coefficient. Consequently, in industrial practice, simpler models are still preferred to avoid such experimental campaign and complex identification process [2]. Therefore, there is a clear demand for new processes of calibration that can simplify the experimental campaign without compromising the accuracy of the models.

Nowadays, there are two main approaches to conduct the identification process: a classical approach and a more recent one based on full-field measurements [3,4]. The classical approach relies on simple tests, that provide near homogeneous strain and stress states over the zone of interest. It is taken advantage of this homogeneity to retrieve the material parameters from simple analytical solutions. This kind of approach has several drawbacks, i.e.: (i) the limited exploitation of experimental tests, since homogeneous stress and strain state assumption can no longer be used after the onset of necking; (ii) the large number of tests required when complex constitutive models have to be calibrated; and (iii) the stress and strain fields do not resemble the ones obtained in forming operations.

The second approach is increasingly being used, mainly because of the rapid development of full-field measurements techniques, such as digital image correlation [5]. These techniques allow a more flexible design of mechanical tests and take advantage of the heterogeneous displacement/strain fields [6]. Indeed, due to the heterogeneity, each material point experiences a different stress and strain history, hence the

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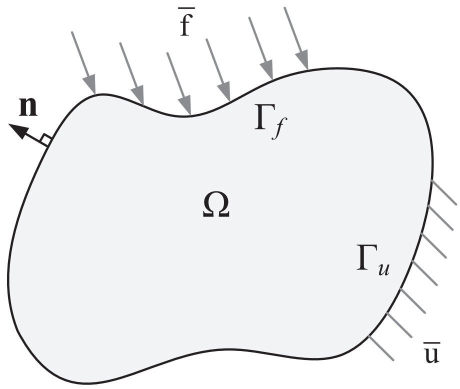

Fig. 1. Domain Ω with prescribed displacement and traction boundary conditions.

number of material parameters governing the field is generally greater than those driving homogenous strain fields [7]. Therefore, this second approach enables to reduce the number of experiments required to calibrate a model. Furthermore, it enables to extend the exploitation limits of a test, since the heterogeneous fields are no longer a problem. However, effective inverse strategies to extract the material parameters from full-field measurements are required.

Accordingly, the development of inverse strategies in computational mechanics has evolved rapidly in recent years, leading to an interesting number of strategies based on full-field measurements, e.g. [3,4,8,9]. The most well-known methods are the Finite Element Model Updating (FEMU) [10], the Constitutive Equation Gap Method (CEGM) [11], the Equilibrium Gap Method (EGM) [12,13] and the Virtual Fields Method (VFM) [14]. These four strategies prove to be effective in identifying parameters associated with linear and non-linear models and, therefore, these will be the focus of this work. However, it should also be mentioned that more strategies have emerged recently with promising results, such as the Constitutive Compatibility Method (CCM) [15], the Dissipation Gap Method [16], the Self-Optimizing Method (Self-OPTIM) [17] and the Integrated Digital Image Correlation Method (Integrated-DIC) [18].

To the best of the authors knowledge, studies on the implementation aspects of these strategies, as well as comparative studies, are rare, specially in elasto-plasticity. Since the mentioned strategies rely on different principles, it is interesting to evaluate their performance in the same conditions, as well as their sensitivity to noise. Thus, the aim of this study is to introduce the four strategies mentioned above, discuss the implementation details and finally, present a comparative study for quasi-static loading conditions. For the sake of simplicity, the scope of this study lies within the framework of infinitesimal small strains. The extension to large strains can be tedious [19,20], with the exception of FEMU, and is out of the scope of this article.

The outline of this work is as follows. A brief description of the inverse problem and the constitutive models used in this study is presented in Section 2. The four inverse strategies selected, FEMU, CEGM, EGM and VFM, are presented in Section 3, as well as flowcharts for each one and a discussion of the main advantages and drawbacks. Finally, in Section 4, the performance of these strategies is compared for two different constitutive models. This performance study starts with a simple case of an isotropic linear elastic model that is afterwards extended for an elasto-plastic model with isotropic non-linear hardening. Moreover, the comparative studies are performed with and without noise.

2. Identification/inverse problem

Consider a continuum solid body whose reference configuration occupies the domain Ω and is bounded by $\partial\Omega$ (see Fig. 1). It is assumed that the material within the domain Ω is homogeneous. The boundary of this body is composed of two sub-boundaries Γ_f and Γ_u, such that $\partial\Omega = \Gamma_f \cup \Gamma_u$ and $\Gamma_f \cap \Gamma_u = \emptyset$. A surface external force is prescribed over Γ_f, possibly with a null value, and a displacement field is prescribed over Γ_u. Neglecting the body forces and assuming static equilibrium, a linear elastic behaviour and infinitesimally small displacements, the mechanical state of the body is governed by three sets of equations: the equilibrium equations,

$$\begin{cases} \mathrm{div}\boldsymbol{\sigma} = 0 & \text{in } \Omega, \\ \boldsymbol{\sigma}\cdot\mathbf{n} = \bar{\mathbf{f}} & \text{on } \Gamma_f, \end{cases} \tag{1}$$

the kinematic compatibility equations,

$$\begin{cases} \boldsymbol{\varepsilon} = \frac{1}{2}\left(\nabla\mathbf{u}(\mathbf{x}) + \nabla^{\mathrm{T}}\mathbf{u}(\mathbf{x})\right) & \text{in } \Omega, \\ \mathbf{u} = \bar{\mathbf{u}} & \text{on } \Gamma_u, \end{cases} \tag{2}$$

and the constitutive equation,

$$\boldsymbol{\sigma} = \mathbf{C} : \boldsymbol{\varepsilon} \qquad \text{in } \Omega, \tag{3}$$

where $\boldsymbol{\sigma}$ denotes the Cauchy stress tensor, $\bar{\mathbf{f}}$ is the prescribed vector of external forces over Γ_f, $\mathbf{u}$ is the displacement vector field, $\bar{\mathbf{u}}$ is the prescribed displacement vector field over Γ_u, $\boldsymbol{\varepsilon}$ is the infinitesimal strain tensor and $\mathbf{n}$ the unit normal vector to $\partial\Omega$.

The stress and strain are related through Eq. (3), for which $\mathbf{C}$ is the constitutive material tensor. It is assumed to be function of a vector that gathers all the unknown constitutive material parameters $\boldsymbol{\xi} = \{\xi_1, \ldots, \xi_n\}$ (n is the number of material parameters). In case of isotropic linear elastic behaviour, $\mathbf{C}(\boldsymbol{\xi})$ represents the Hooke's elasticity tensor and $\boldsymbol{\xi}$ contains two parameters: Poisson's ratio ν and Young's modulus E, $\boldsymbol{\xi} = \{\nu, E\}$, respectively.

For the direct problem of continuum mechanics, the initial shape of the solid body, the material parameters and the set of boundary conditions, $\bar{\mathbf{f}}$ and $\bar{\mathbf{u}}$, are assumed to be known. Accordingly, the unknowns are the fields $(\mathbf{u}, \boldsymbol{\varepsilon}, \boldsymbol{\sigma})$, which must satisfy the three previous sets of Eqs. (1), (2) and (3). For the inverse problem of parameter identification using full-field measurements, the aim is to retrieve the material parameters given a discrete observation of the displacement field $\hat{\mathbf{u}}$ and information concerning the boundary conditions, $\bar{\mathbf{f}}$ and $\bar{\mathbf{u}}$. The measured displacement field $\hat{\mathbf{u}}$ can be obtained, for instance, through a non-contact measurement technique, such as DIC, and the strain field required to calculate the stress field can be calculated using Eq. (2). The idea behind the inverse problem is to explore an implicit relationship between the measured displacement field and the parameters of the constitutive model.

Typically, full-field measurements are performed on the surface of the body and this limits the identification through the volume. Therefore, the inverse problem in linear and non-linear cases is usually seen as a in-plane problem, for which the plane stress assumption can be adopted. This assumption implies that the body with domain Ω is a thin flat body, with volume V and a constant thickness t that is assumed much smaller than the other dimensions. Furthermore, the body only undergoes in-plane loading.

For the case of non-linear elasto-plastic behaviour, the linear relationship between stress and strain is no longer valid, and the constitutive equations are obtained within the classical incremental theory of plasticity. In the following, these equations are briefly recalled.

Consider the additive decomposition of the total strain tensor increment $d\boldsymbol{\varepsilon}$, in terms of elastic $d\boldsymbol{\varepsilon}^{\mathrm{e}}$ and plastic $d\boldsymbol{\varepsilon}^{\mathrm{p}}$ components, which can be written as

$$d\boldsymbol{\varepsilon} = d\boldsymbol{\varepsilon}^{\mathrm{e}} + d\boldsymbol{\varepsilon}^{\mathrm{p}}. \tag{4}$$

Moreover, consider an hypoelastic relationship to describe the stress-strain relation, as follows

$$d\boldsymbol{\sigma} = \mathbf{C} : \left(d\boldsymbol{\varepsilon} - d\boldsymbol{\varepsilon}^{\mathrm{p}}\right), \tag{5}$$

where $d\boldsymbol{\sigma}$ is the stress increment. The plastic strain increment $d\boldsymbol{\varepsilon}^{\mathrm{p}}$ can be defined by means of three key concepts: a yield criterion, a hardening