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Shakedown boundaries of multilayered thermal barrier systems considering interface imperfections

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A R T I C L E I N F O

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A B S T R A C T

Thermal barrier coatings (TBCs) usually have complex structures and operate in cyclic high temperatures. The cracking phenomenon is usually observed at the interface with microscopic imperfections between the bond and the thermally grown oxide (TGO) layer owing to the cyclic plasticity around the defect interface under the cyclic thermal loading. Shakedown limits of TBCs with interface imperfections are investigated based on the proposed linear matching method considering the effect of temperature-dependent material properties. The influences of geometric parameters h_o, R/h_o and H/R as well as the thermal expansion coefficient ratio α/α_r are discussed in detail. Results present that the thickness of the TGO has no effect and the geometric factor R/h_o shows a little influence on shakedown limits of the multilayered systems. However, the geometric parameter H/R and the thermal expansion coefficient ratio α/α_r have a remarkable impact on shakedown limits. This indicates that the depth of the defect influences shakedown limits greatly, while the transition radius at the corner of the imperfection shows slight effect. This is obviously different from the common phenomenon that the transition radius usually has an important influence on shakedown limits. Based on the calculated data, the shakedown assessment approach of TBCs considering various factors mentioned above is proposed. It is of great interest that the simulated effective plastic strain occurs at the defect interface between the TGO and the bond layer when the applied load exceeds the shakedown limit. It is the crack failure region of cracked TBCs according to the microscopic morphology observation.

1. Introduction

Until now, hundreds of different types of coatings are developed to protect various engineering structures from wear, corrosion, erosion, thermal failure and so on. In these coatings, TBCs are usually used for complex structures which operate in very high temperature condition, such as turbine and combustor engines [1].As a multilayered system, typical TBCs are composed of four layers, including TBC itself, the superalloy substrate, the bond layer between the TBC and the substrate, and TGO produced between the bond layer and the TBC. Generally, extremely large stresses are always produced due to the thermal expansion mismatch between different layers. Therefore, the stress distribution of multilayered systems is a key issue for the safety assessment, especially at high temperature. So far, many works have been performed for the stress distribution of multilayered systems operating at high temperature. Chen et al. [2,3], Limarga and Wilkinson [4] and Li et al. [5] deduced the time-dependent creep deformation and stress state of multilayered systems considering the elastic-creep condition. Additionally, Mahbadi et al. [6] introduced the linear Prager kinematic hardening model, and Nakane et al. [7] considered the non-linear kinematic strain-hardening model to describe the cyclic stress-strain relationships under elastic-plastic-creep conditions. However, the interface imperfections, which have a great effect on the cracking or delamination of multilayered systems, have not been considered by researchers above. Huang and He [8,9] researched the stress distribution of multilayered systems considering the effect of imperfect interfaces. Mumm et al. [10] found that the main cracking factor of TBCs is the ratcheting deformation near the interface imperfections under cyclic thermo-mechanical loads, as shown in Fig. 1. Karlsson and Evans [11] further considered that the plastic zone around the interface imperfections of TBCs depends on the temperature cycling. Gralewicz et al. [12] Ptaszek et al. [13] and Netzelmann and Walle [14] developed the active thermography technique to investigate the delamination behaviors at the interface of multilayered systems, respectively. These works are very helpful for explaining the failure mechanism and establishing the safety assessment approach for TBCs under repeated thermal loads. However, according to the best knowledge of authors, very few investigations on the shakedown boundaries of multilayered systems have been addressed, especially for the TBCs with interface imperfections. It is known that the shakedown limit is an important index to guard against the accumulated plastic deformation or ratcheting effect of structures under cyclic loads [15–17].

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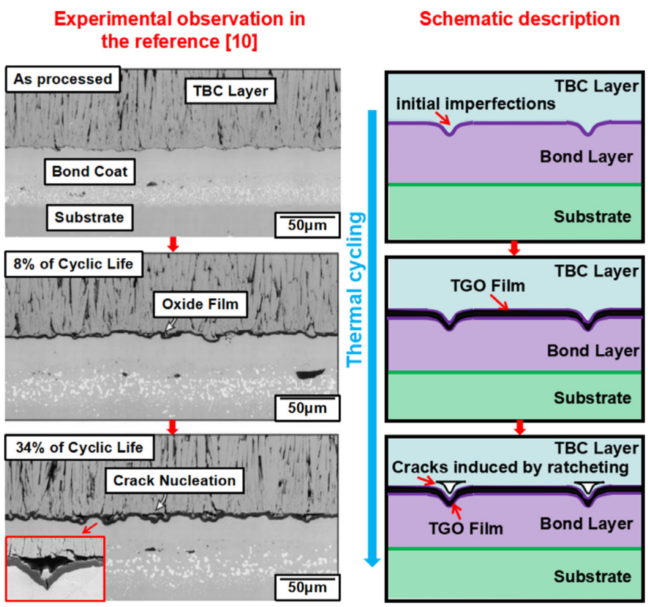

Fig. 1. Damage mechanism of TBCs induced by ratcheting deformation.

Accordingly, it is highly necessary to discuss the shakedown limits of multilayered TBCs with interface imperfections for the safety evaluation.

In this work, the linear matching method based on the kinematic theory is utilized to analyze shakedown limits of multilayered TBCs with interface imperfections. The influences of key geometric parameters and thermal expansion coefficients are investigated systematically to discuss the shakedown assessment approach for engineering applications. Additionally, the temperature-dependent yield stress of the bond layer, which impacts the shakedown limit of structures greatly, is considered based on the experimental data to produce the shakedown boundaries of TBCs subjected to cyclic temperature loads.

2. Basic theory for shakedown limit analysis by the linear matching method

Generally, it is hard to assess the ratcheting and shakedown limits due to the complexity of classical shakedown theory and geometrical structures. One typical estimation method for complex structures in EN13445 is to simulate the cyclic elastic-plastic strain responses directly over 500 cycles [18]. To decrease the numerical expense, a direct cyclic analysis method is proposed and incorporated into the ABAQUS by Nguyen-Tajan et al. [19]. However, a large number of simulations should be performed under various load histories to assess the ratcheting and shakedown limits approximately. As an alternative approach, the shakedown limit can be calculated directly based on the classical static or kinematic theorem. This direct analysis method is considered to be better than the previous step-by-step analysis due to its high efficiency. Recently, the direct analysis approaches have been developed rapidly, such as the linear matching method [20,21], the non-cyclic method [22,23], the elastic compensation method [24], the generalized local stress strain τ-node method [25] and some other methods [26–28]. In which, the linear matching method based on the kinematic theory of Koiter has been verified to accurately calculate the upper bound shakedown boundaries with high efficiency [29].

According to the linear matching method, it is postulated that the material of the bond layer is elastic-perfectly plastic and meets the von Mises yield condition, the TGO layer and the substrate is totally linear-elastic. Taking account of the cyclic high temperature cases of multilayered systems, the temperature-dependent yield stress of the bond layer, $\sigma_y^{bc}(T)$, is considered. To achieve the temperature-dependent yield stress varying with the temperature T, the yield stress $\sigma_y^{bc}(T)$ is updated during the iterative calculation process.

Assume a cyclic temperature field $\lambda\theta(x_i,\ t)$ operating within the whole inner body and the surface loads $\lambda P_i(x_i,\ t)$ acting on the specified area S_T are applied. Where, λ represents a load factor, which is used to calculate all of the load histories. On the surface without surface loads, S_r, the displacement equals zero. Moreover, if the elastic thermal stress histories $\tilde{\sigma}_{ij}^{\theta}$ under temperature loads $\theta(x_i,\ t)$ and the elastic mechanical stress histories $\tilde{\sigma}_{ij}^{P}$ under surface loads $P_i(x_i,\ t)$can be solved, the linear elastic stress histories of the structure under combined thermal-mechanical loads can be calculated by the superposition principle:

$$\lambda\tilde{\sigma}_{ij} = \lambda\tilde{\sigma}_{ij}^{\theta} + \lambda\tilde{\sigma}_{ij}^{P} \tag{1}$$

Under cyclic temperature loads, the stress histories during a typical cycle $0 \le t \le \Delta t$ are

$$\sigma_{ij}(x_i, t) = \lambda\tilde{\sigma}_{ij}(x_i, t) + \bar{\sigma}_{ij}(x_i) + \bar{\sigma}_{ij}^{r}(x_i, t) \tag{2}$$

Here, $\bar{\sigma}_{ij}^{r}(x_i, t)$ represents the residual stress components changing with time, and $\bar{\sigma}_{ij}^{r}(x_i)$ is a constant residual stress in equilibrium with external surface loads on S_T. It should be noted that $\bar{\sigma}_{ij}^{r}(x_i, t)$ is equal to zero for the shakedown condition. Therefore, the cyclic stress field under the shakedown condition becomes

$$\sigma_{ij}(x_i, t) = \lambda\tilde{\sigma}_{ij}(x_i, t) + \bar{\sigma}_{ij}(x_i) \tag{3}$$

Taking into account the temperature-dependent yield criterion, the relationship between the strain rate $\dot{\varepsilon}_{ij}^{i}$, the shear modulus G and the temperature-dependent yield stress is

$$\sigma_y^{bc}(T) = \frac{3}{2}G\bar{\dot{\varepsilon}}^{i} \tag{4}$$

where, $\bar{\dot{\varepsilon}}^{i}$ is the equivalent strain rate, and $\bar{\dot{\varepsilon}}^{i} = \sqrt{\frac{2}{3}\dot{\varepsilon}_{ij}^{i}\dot{\varepsilon}_{ij}^{i}}$.

For a given shear modulus G, the incompressible relationship at each instant is proposed under a constant residual stress field $\bar{\sigma}_{ij}^{f}$ in a cycle as follow

$$(\dot{\varepsilon}_{ij}^{f})' = \frac{1}{G(t)}(\lambda_{ub}^{i}\tilde{\sigma}_{ij} + \bar{\sigma}_{ij}^{f})', \quad \dot{\varepsilon}_{kk}^{f} = 0 \tag{5}$$

where, the load factor λ_{ub}^{i} is the upper bound for $\dot{\varepsilon}_{ij}^{i}$. The constant residual stress field $\bar{\sigma}_{ij}^{f}$ can be calculated by integrating Eq. (5) over the whole cycle. Accordingly, the increment of plastic strain over the cycle $\Delta\varepsilon_{ij}^{f}$ and $\bar{\sigma}_{ij}^{f}$ has the following linear relationship:

$$\Delta\dot{\varepsilon}_{ij}^{\prime f} = \frac{1}{\bar{G}}\left(\sigma_{ij}^{\prime in} + \bar{\sigma}_{ij}^{\prime f}\right), \Delta\varepsilon_{kk}^{f} = 0 \tag{6}$$

Here, $\sigma_{ij}^{\prime in} = \bar{G}\{\int_0^{\Delta t}\frac{1}{G(t)}\lambda_{UB}^{i}\sigma_{ij}^{\prime}(t)dt\}$ and $\frac{1}{\bar{G}} = \int_0^{\Delta}\frac{1}{G(t)}dt$.

Therefore, the upper bound on the shakedown limit is obtained by

$$\lambda_{UB}^{f} = \frac{\int_V\int_0^{\Delta t}\sigma_y^{bc}(T)\cdot\bar{\dot{\varepsilon}}\cdot\dot{\varepsilon}_{ij}^{f}dtdV}{\int_V\int_0^{\Delta t}\tilde{\sigma}_{ij}\cdot\dot{\varepsilon}_{ij}^{f}dtdV} \tag{7}$$

The minimum upper bound can be achieved by repeating the above process. To analyze the shakedown boundaries of multilayered TBCs with complex imperfections easily, the above numerical algorithm is implemented into the general commercial finite element software ABAQUS [30] with UMAT user subroutine. The iterative algorithm assesses a varying shear modulus G by matching the stress produced by the linear model proposed above and the temperature-dependent yield criterion at the strain rate $\dot{\varepsilon}_{ij}^{i}$ is calculated by the previous iterative step.

To describe this approach clearly, the iteration procedures are illustrated as follows:

(1) For an iteration number $k = 1$, we define $\tilde{\sigma}_{ij}(t_n) = \tilde{\sigma}_{ij}(t_n)_{ext}$ for n vertices of the loading history and $\bar{\mu} = 1$. The elastic stresses $\tilde{\sigma}_{ij}(t_n)_{ext}$ with respect to n vertices of the loading history can be obtained separately based on calculating a linear problem.