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# Extracting elastic-plastic properties from experimental loading-unloading indentation curves using different optimization techniques

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A B S T R A C T

This work is focused on the determination of elastic-plastic material properties from indentation loading-unloading curves using optimisation techniques and experimental data from instrumented indentation tests. Three different numerical optimisation methods (namely, FE analysis, dimensional mathematical functions and simplified mathematical equations approaches) have been used to determine three material properties; Young's modulus, yield stress and work-hardening exponent. The predictions of the material properties from the three approaches have been validated against the values obtained from uniaxial tensile tests and compared to the experimental loading-unloading curves. In general, the elastic-plastic material properties predicted from these three proposed optimisation methods estimate the Young's modulus to within 6% and the yield stress and work-hardening exponent to within 12%, compared to the values obtained from the uniaxial tensile tests.

## 1. Introduction

Indentation techniques have been used for mechanical characterisation of materials for decades due to their non-destructive nature and applicability to small sized samples. Fig. 1 shows a schematic illustration of an indentation testing system [1] where a downward load is applied to the indenter to penetrate the test sample, and the reaction force and the displacement at the indenter tip are recorded during the test. Different approaches have been proposed to obtain the mechanical material properties, such as Young's modulus (E), yield stress ($\sigma_y$) and work-hardening exponent (n), from the indentation data, see e.g. [2–12]. In many studies, there is only one interpreting method involved and it is usually performed using numerical simulations, see e.g. [4,8]. Experimental indentation tests have been carried out for different materials using different indenter geometries and compared to the corresponding numerical simulations, e.g. [6,7,12].

To analyse the material response of an indented specimen, the effects of the indenter geometry on the prediction of the material properties have been investigated by Kang et al [13] using the commercial FE software ABAQUS with new optimisation approaches combining three different methods: (i) Combined FE Simulation and optimisation [14] (ii) Combined dimensional analysis and optimisation [15] and (iii) Optimisation using simplified equations [16]. However, the previous optimisation techniques [14–16] have been mainly based on simulated target FE loading-unloading curves, rather than curves obtained from experimental tests. It has been found in a previous study [14] that determining elastic-plastic properties from indentation data using only FE simulation and optimisation is less accurate when it is based on experimental indentation data with random errors. Therefore, it is worth extending the investigation to the other two developed optimisation approaches to evaluate their feasibility and robustness.

This study highlights the extraction of elastic-plastic properties from experimental instrumented indentation loading-unloading curves, using the three developed optimization techniques. The general performance and the applicability of these techniques are evaluated and some limitations and areas that need to be explored in the future are addressed. In this study, the experimental loading-unloading curves are obtained using a single Berkovich indenter under different indentation loads [14].

To investigate the mechanical properties of materials that exhibit a power law hardening, which is generally assumed to characterise the work-hardening plasticity behaviour of metals including steels, the stress-strain relationship is given as follows:

$$\sigma = \begin{cases} E\varepsilon, & \varepsilon \le \frac{\sigma_y}{E} \\ K\varepsilon^n, & \varepsilon > \frac{\sigma_y}{E} \end{cases} \quad (1)$$

where the coefficient K is given by:

$$K = E^n {\sigma_y}^{1-n} \quad (2)$$

## 2. Nanoindentation and tensile experimental data

Room temperature nanoindentation tests with a Berkovich indenter have been performed in [17] on P91 steel specimens with maximum loads of 150 mN, 200 mN as shown in Fig. 2. Ten indentation tests

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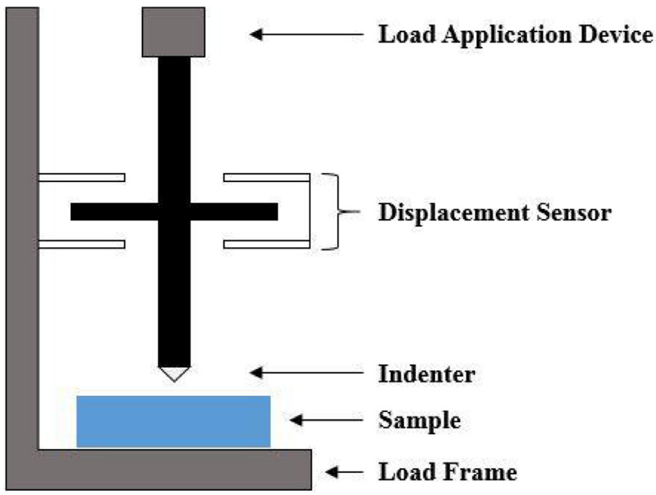


**Fig. 1.** Schematic illustration of a typical instrumented indentation system [1].

have been completed at each load level to provide accurate indentation curves. The details of the nanoindentation tests are presented in Table 1 where the loading time and the unloading time were set at 20 s and 10 s respectively. Young's modulus of the P91 steel at room temperature can be obtained based on the Oliver-Pharr method which uses the unloading part of the indentation curve to obtain Young's modulus [19]. The average values of Young's modulus at each load level are presented in Table 2.

**Table 1**
Details of the nanoindentation tests.

| Material | Geometry of indenter | Temperature | Applied force |
|---|---|---|---|
| P91 steel | Berkovich indenter | Room (23 ○C) | (100 mN, 150 mN and 200 mN) |

**Table 2**
Young's modulus from nanoindentation tests based on the Oliver-Pharr method.

| Load (mN) | 100 | 150 | 200 |
|---|---|---|---|
| Young's modulus (GPa) | 251 | 253 | 244 |

Uniaxial tensile tests at room temperature (23 ○C) on P91 steel specimens have also been performed [18] to obtain the stress-strain uniaxial data. P91 true stress-true strain curve is shown in Fig. 3 where Young's modulus (E) is 215 GPa and the yield stress ($\sigma_y$) is 515 MPa at a strain of 0.0033. A power law hardening, described by Eqs. (1) and (2), was assumed for the plasticity of the material. By fitting the stress-strain data from Fig. 2, the hardening exponent n was determined to be 0.136. The material properties obtained from the uniaxial tensile stress-strain data can be used to validate the optimised results based on the three different optimisation techniques. It is interesting to note that Young's modulus values for P91 steel obtained from the nanoindentation tests using the

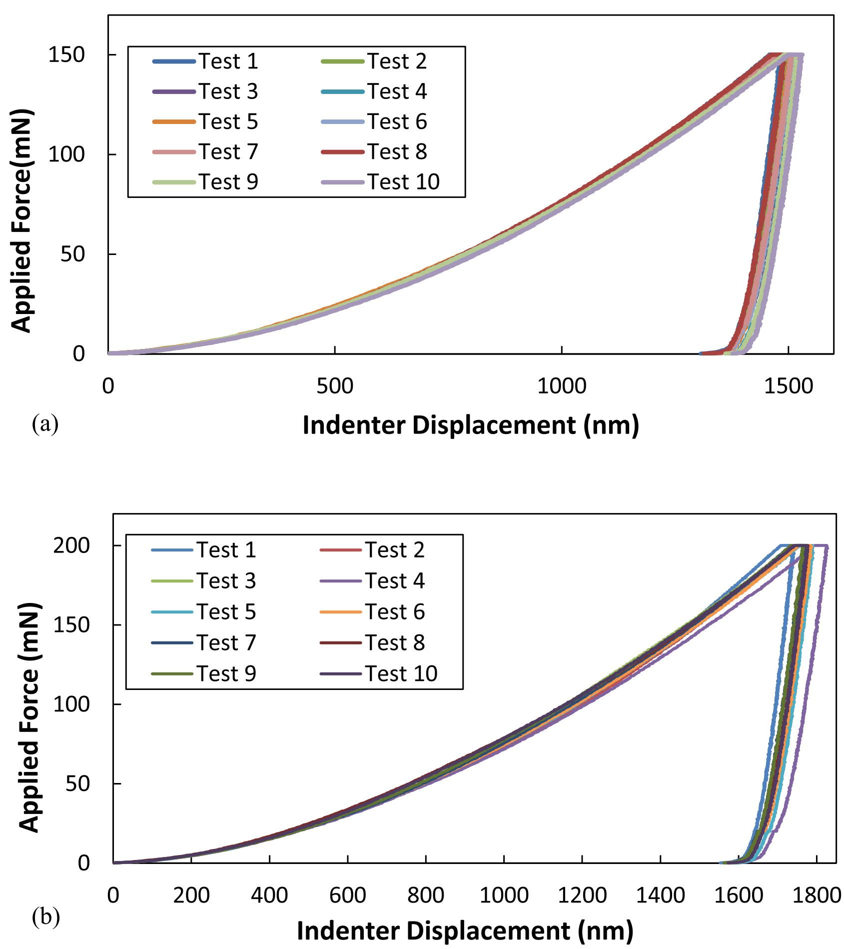


**Fig. 2.** Experimental loading-unloading curves at (a) 150 mN and (b) 200 mN load levels using a Berkovich indenter [17].