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Effects of free-edge interface angle on bi-material shear strength

Robert C. Wetherhold*, Gary F. Dargush, Tapan S. Mhatre

Department of Mechanical and Aerospace Engineering, University at Buffalo, SUNY, Buffalo, NY 14260-4400, United States

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ABSTRACT

The bonded interface between dissimilar materials is often the site of premature failures, especially when there is a stress singularity at the intersection of the interface and the free edge or surface. However, the singularities, which depend upon the elastic constants and the interior angles of the two materials, may vanish for certain ranges of these angles. Thus, proper selection of the interior angles offers the possibility of enhancing joint strength. In this work, the focus is on the shear strength of bonded aluminum-epoxy cylinders of uniform diameter under torsional loading and the effect of selecting angles for which the stress singularity at the edge disappears. A new boundary element framework is developed for pure shear loading to determine the power of the singularity for general interface angles and the value of the associated generalized stress intensity factor (GSIF), using a weighted traction axisymmetric torsional mode formulation. Physical experiments are conducted to examine whether changes in the interior angles produce differences in measured joint strength and failure patterns. Three different geometries are considered: a “concave” angle of 37° for the aluminum cylinder, a butt joint with 90° angles for both cylinders and a “convex” aluminum cylinder with an interior angle of 143°. Based upon elastic theory, the first two cases are associated with singularities, while the third is singularity-free. The experiments demonstrate that the joint shear strength can be greatly improved by avoiding stress singularities through the selection of a proper edge angle. From a physical perspective, this strength improvement is due to crack tortuosity, extensive adherence of epoxy to aluminum, and epoxy matrix spalling. Low strength specimens show rapid debonding and minimal adherence.

1. Introduction

The durability of a composite structure often relates directly to the adhesive bond strength between two dissimilar constituent materials. Unless precautions are taken, initial cracks may grow, leading to premature and perhaps catastrophic failure. The usual approach is to over-design the joints, using available strength data (often with large data scatter) combined with empirical “safety factors”. Naturally, this approach comes with a penalty, in terms of both weight and cost. However, an alternative approach can be taken by considering the effects of local geometry at the intersection of the interface with the free surface, where failure usually initiates. At these locations, depending upon the material properties and the interior angles, elasticity theory may predict stress singularities even without the presence of interface edge cracks. While these unbounded stresses provide an underlying cause for premature failures, the theory also suggests a means for potential improvement of joint strength. By understanding the local stress distributions, alternative geometries can be investigated to seek designs that avoid interface stress singularities. Consequently, this approach could lead to improvement in joint strength.

Initial results for adhesive strength under tension (Mode I fracture) were achieved by Wang and Xu [1], where the external surface near the interface along the free edge was disrupted by introducing local convex protuberances. Increases in strength of approximately 20% were achieved for aluminum-PMMA joints under quasistatic and dynamic physical experiments compared to the average strength for corresponding butt joint specimens. Unfortunately, this geometric design configuration disrupts the otherwise smooth profile of the external surface of the joint, which may not be desirable or even possible in most situations. More recent work by Xia et al. [2] and Wetherhold and Dargush [3] demonstrated improvements in adhesive joint strength, while retaining a smooth outer surface by adjusting the internal joint angle for aluminum-epoxy cylinders. In the former paper, Xia et al. [2] use a finite element method to analyze particular angle configurations aimed at minimizing interface stresses. The resulting improvements in overall strength are encouraging, but the finite element approach does not lend itself to a generalized design process. Additionally, their effort to minimize interface stresses actually pushes the design to interior angles that are at the very cusp of the singular zone, which would suggest that those designs may be quite sensitive to geometric imperfections. On the other hand, Wetherhold and Dargush [3] take a more holistic view by investigating the singular characteristics over a broad range of interior angles under tensile loading. Strength data from physical experiments for three different angles are found to be consistent with the expected order based

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* Corresponding author.
E-mail address: mecrcw@buffalo.edu (R.C. Wetherhold).

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upon the associated powers of the interface stress singularities, which are determined by solving a local interface eigenproblem.

For a variety of joint configurations, the applied loads involve not only adhesion (Mode I), but also shear (Mode II). For example, failure of a joint between two rods which are transmitting torque primarily involves Mode II. The previous results by Lauke et al. [4,5] for adhesive joints were achieved by solving for internal interface angles to eliminate or decrease the generalized stress intensity factor. Here, an entirely new approach will be developed for Mode II by using an axisymmetric boundary element framework for torsional response to determine the power of the singularities and to estimate the corresponding generalized stress intensity factor (GSIF). Physical experiments are then conducted under pure torsional loading for cylindrical aluminum-epoxy bi-material geometries, having three different interior angles at the interface-free edge intersection, classified as concave, flat and convex from the vantage point of the aluminum cylinder. Strength data and failure modes are then compared across the different geometries and also to the underlying theory and computations. Finally, the system is scaled to investigate size-effects with and without singular interface stresses.

The remainder of the paper is organized as follows. Section 2 discusses the mechanics of bi-material interfaces under torsional loading. Included are details of the weighted traction axisymmetric boundary element method, the analytical formulation of the interface eigenproblem, the solutions for the stress singularity exponents and the corresponding GSIFs. Section 3 defines the materials and specimen fabrication, while the results of the physical experiments are presented in Section 4. Afterwards, Section 5 provides a discussion and several conclusions.

2. Bi-material interface mechanics for torsional loading

2.1. Theoretical and experimental background

The field of linear elastic fracture mechanics (LEFM) has a long history of development, beginning from the ideas of Griffith [6] and the mathematical work by Kolossoff [7], Muskhelishvili [8] and Irwin [9]. For sharp cracks within the LEFM idealization, the stresses near the tip approach an infinite value as $r^{-\beta}$ with r representing the distance from the crack tip and $\Re(\beta) = 1/2$. Williams [10] defined in detail the stress distribution in the vicinity of the crack tip in these non-smooth problems that contain stress singularities. Stress, of course, cannot be used as a proper measure in such problems and, instead, a stress intensity factor (SIF) is introduced. The SIF can be related to the energy release rate, thus establishing a criterion for crack propagation (e.g., Anderson [11]).

In a more general setting, stresses can become unbounded without the presence of a crack. Typical situations include sharp notches and the intersection of bi-material interfaces with free surfaces. For these cases, the stresses again behave as $r^{-\beta}$ close to the critical singular location, with r representing the distance from the singular point, as illustrated in Fig. 1 for the case of two fully bonded cylinders. However, for such cases, $\Re(\beta) < 1/2$. As a result, a bounded generalized stress intensity factor (GSIF) can be defined, although the direct connection to an energy criterion remains to be established. Important work on the bi-material interface and GSIF problems includes that by Williams [12], Bogy [13,14], Hutchinson et al. [15], Rice [16], Carpinteri [17], Reedy and Guess [18–22], Dunn et al. [23–25], Lauke et al. [4,5], Lim et al. [26], Dargush and Hadjesfandiari [27, 28], Campillo-Funollet et al. [29] and Wetherhold and Dargush [3], among others. The focus of this previous research has been primarily on axial tension (mode I) loading.

In the current work, the focus is on the pure torsional elastic response of bi-material cylindrical rods and the shear stress singularities that may be present at the intersection of the interface and the free outer surface of the perfectly bonded cylinders. This reduces to a scalar axisymmetric problem involving only the zeroth anti-symmetric mode written in terms of the circumferential displacement component. As a result, simplifications which allow the development of some interesting new analytical and computational treatments are possible.

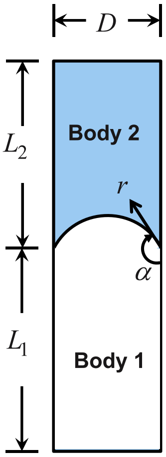

Fig. 1. Bonded cylinders.

2.2. Axisymmetric boundary element framework

To understand the behavior of interfacial stresses near the free surface of bi-material isotropic elastic cylinders under torsion, a multi-region axisymmetric boundary element methodology (BEM) is developed. The first objective of this development is to estimate the power of the stress singularity at the free edge on the interface between the two cylinders. This is accomplished by using maximum principal stress results from a series of h-refined boundary element analyses. The second objective is to provide an estimate of the generalized stress intensity factor (GSIF) associated with that same location for torsional loading. For this task, a weighted traction axisymmetric boundary element formulation is developed to provide a convergent algorithm for these bounded GSIF-related quantities. To the authors' knowledge, this is the first presentation of such a framework.

In particular, this newly developed, comprehensive computational framework enables the study of generalized stress intensity factors for bi-material cylinders under pure torsional loading, based upon the boundary element method [30,31]. The framework includes a methodology to estimate the power of the stress singularity on the interface at the free surface and a capability to extract a bounded generalized stress intensity factor, all using anti-symmetric mode 0 axisymmetric boundary integral representations, as detailed below.

For this development, the geometry of each cylinder is considered to be a purely axisymmetric body with diameter D, occupying the domain Ω in the R–Z plane with outer boundary Γ and the Z-axis as the axis of symmetry. Fig. 2 provides a schematic for the problem.

The pure torsional response can be captured as the zeroth anti-symmetric mode, for which the governing differential equation can be written in (R, Θ, Z) cylindrical coordinates as

$$G\left[\frac{\partial^2}{\partial R^2} + \frac{1}{R}\frac{\partial}{\partial R} - \frac{1}{R^2} + \frac{\partial^2}{\partial Z^2}\right]u_\Theta + f_\Theta = 0 \tag{1}$$

with the circumferential displacement u_Θ as the dependent variable. In (1), G represents the elastic shear modulus, assumed constant within each cylinder, while f_Θ is the applied torsional body force density.

The corresponding boundary integral representation can be developed by multiplying (1) by a circumferential displacement function U_Θ, integrating over the domain, and applying integration by parts and the divergence theorem to shift all spatial derivatives from u_Θ to U_Θ. Then,