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Analysis of stator vibration response for the diagnosis of rub in a coupled rotor-stator system

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ABSTRACT

The present day rotors are more susceptible to rub due to reduced clearance that adversely affects the performance of the machines. In most of the theoretical investigations on rub, the structural connectivity between rotor and the stator casing is ignored and the dynamics of the rotor are studied with stator interaction as external to the system. Thus, in the present study, a coupled rotor-stator system is modelled where the vibrating rotor transmits the force to the stator structure through the bearing support and during the direct contact. The rub interaction is defined using contact mechanics based Lagrange multiplier method. The model ensures dynamic rotor-stator contact boundary and more realistic contact constraints in contrast to most of the earlier approaches. The stator thus responds through its own vibratory motion and the instantaneous clearance decides the nature of interaction between them. The stator vibration data at the casing location is explored to provide conclusive indication of the rub phenomenon. The intermittent rub interactions that are non-synchronous to rotational frequency cause the system to exhibit quasi-periodic response and the transient excitation due to impacting rotor generates natural frequencies in the system response. In the stator vibration spectrum, the presence of sub harmonic frequency components and stronger presence of harmonics of rotational frequency near the natural frequencies of the system indicate the occurrence of rub. Only low amplitude rotational frequency component is observed otherwise. Strongly correlated with rotor vibration response, the stator vibration response and its rub specific features are proposed for the rub diagnosis.

1. Introduction

In any rotating machinery, the rotor is supported on bearings that are connected to the support structure. The non-rotating support structure surrounding the rotor is known as stator. The efficiency of a machine such as turbine, compressor etc. is greatly improved by minimizing the clearance between the rotor and the stator. This makes the rotor more susceptible to rub in case of its passage through critical speed or any other factor that cause an increase in vibration amplitude. For a practical rotor assembly where stator is structurally connected and vibrates during rotor operation, it would be essential to consider the dynamic response of both the rotor and the stator for exploring the overall system dynamics of rub rather than considering only the rotor response.

Vibration characteristics of various rotor systems (Jeffcott rotor, jet engine rotors, turbo- machineries, etc.) under rubbing condition have been investigated extensively in the past. With theoretical and experimental works, techniques and strategies to extract rub related features such as sub-harmonics and super-harmonics of rotational frequency have been explored [1,2]. A mathematical model for rub identification that indicates the presence of mild rubbing and the onset speed of instability in the spectral signature of the response is reported [2]. Presence of second and third harmonics of the synchronous frequency is associated with rub however, it was emphasised that the entire spectrum should be monitored and the tracking of only the synchronous components may lead to incorrect interpretations. In a wind mill imbalance problem [3] super-harmonics of order 9 and sub-harmonics of order up to 32 are also observed. The modelling and investigation approaches for rub phenomenon have been reviewed [4–7] in detail. The need for modelling practical turbo-machineries and identification of major influencing parameters on the dynamic response are emphasised [5]. The dynamic characteristic of blade-casing rubbing for coated and non-coated casing has been reviewed thoroughly [6]. With the improved computational capabilities in recent times, it is possible to have more interactive rub models and reliable solution techniques.

An important aspect of rub analysis is its nonlinear behaviour that has been explored in different ways by several researchers in the past [8–10]. An analytical model of a Jeffcott rotor for rotor casing rub interaction is solved and the rub forces, energy levels, rub duration and overall orbital response is discussed [8]. The effects of system parameters (casing stiffness, unbalance, friction and damping) on the rub related fea-

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Nomenclature

c_g	$=g/\max(X(\omega))$, clearance ratio
$\mathbf{C}$	effective damping matrices
CC_{lag}	cross-correlation coefficient in terms of data point lag
$\mathbf{D}$	damping matrix of the system
$\mathbf{f(t)}$	global force vector of body forces and surface traction
g	initial gap between contacting bodies
g_N	gap function in normal direction
$\mathbf{G}_\mathrm{N}$, $\mathbf{G}_\mathrm{T}$	global contact matrix for normal and tangential direction
$\mathbf{G}_\mathrm{NT}$	$=\mathrm{GN}+\mu\mathbf{G}_\mathrm{T}$, global normal-tangential contact matrix
k_s	stiffness of the chosen stator
k_{ref}	reference stator stiffness
$\mathbf{K}$	stiffness matrix of the system
$\mathbf{M}$	mass matrix of the system
N_r	no. of data points
Q	total no. of data points in time series
r	$=\omega/\omega_{\mathrm{n1},}$ speed ratio
r_d	radius of the disc
$\mathbf{u}^*$	displacement update without constraint
$\mathbf{u}$, $\dot{\mathbf{u}}$, $\ddot{\mathbf{u}}$	displacement, velocity and acceleration vector
$\mathbf{u_c}$	corrective displacement vector
$X_p,\ Y_p$	p^{th} element of time data series X and Y
$\overline{X}$, $\overline{Y}$	average of time data series X and Y
$X(\omega)$	response as function of speed
$\omega\mathbf{G}$	gyroscopic matrix of the system
$\square_r$, $\square_s$	subscript 'r' and 's' stands for rotor and stator respectively
B	$=k_s/k_{ref}$, ratio of chosen stator stiffness to reference stiffness
λ_N, λ_T	normal and tangential contact force
μ	friction coefficient
Ω	speed of rotation
ω_{n1}, ω_{n2}, $\omega_{\theta1}$	natural frequencies; first bending, second bending and first torsional

tures are also studied. The rotor response during contact between rotor and stator exhibits alternate periodic, quasi-periodic and chaotic vibrations as the rotating speed increases [9,10]. Very rich forms of periodic and chaotic vibrations are found in an experimental work along with the presence of fractional harmonic components such as $\frac{1}{2}\mathrm{X},\frac{3}{2}\mathrm{X},\frac{5}{2}\mathrm{X}$ [11]. The influence of random stiffness on a rubbing rotor under random excitation causes the nonlinear response to dominate in subcritical regions and suppressed in the supercritical speed regions [12]. A disk-drum rotor system with rub impact exhibits full annular rub with period-1 motion as the speed increases. Periodic and quasi periodic motion is found to appear alternatively for certain range of speed [13]. Sub-harmonic resonance occurs in an aircraft rotor due to maneuver load that causes the rub impact [14]. The presence of quasi-periodic motion is observed due to the small damping considered in the simulation model.

The response characteristics of a dual rotor system under rub impact exhibits combination frequency components [15]. The parametric study indicates the significance of mass eccentricity and the rotational speed whereas the inter-shaft stiffness is found insignificant. Yang et al. [16] analysed a dual rotor system simulating aero-engine for fixed point rubbing and observed many combination frequencies. The simulation model results are validated with experimental results. In a blade-casing rub phenomenon the torsional vibration features are found to be more significant in the shaft-disk-blade system. Large stator stiffness and high rotational speed may cause period-2 motion. The response amplitude of the rotor torsional vibration and blade bending vibration tend to increase with increase in casing stiffness [17].

In order to avoid untimely failures, health assessment of rotors under operation becomes an integral part of the research and development. Edward et al. [18] provided a broad review on the fault diagnosis techniques with regard to rotating machinery. The rotor faults such as mass unbalance, misalignments, shaft bow and shaft cracks are given more emphasis. In order to distinguish rub from other rotor faults that generates similar spectral features, the directional nature of rub and the shift in resonance speed are explored [19]. Various signal processing techniques such as Hilbert-Huang transform (HHT), full spectrum and empirical mode decomposition (EMD) are used for rub diagnosis [19–21].

Recently, contact mechanics based approaches are adopted to model the rotor-stator interactions [22–25]. The rotor speed transients due to rub caused by accidental imbalance in a turbo-generator is investigated. Lagrange multiplier method [22,23,25] and augmented Lagrange method [24] is adopted for investigation of rotor-stator interaction. In a two disc rotor system under different loading conditions, the rubbing excites quasi periodic motion and the response spectrum exhibits rotational harmonic and the system natural frequency [26]. In a multilateral contact, it was found that the four-point rubbing reduces the hazard of full annular rub and restrains sub-synchronous vibration. At higher operational speed, four pin stator arrangement exhibits more stable vibration response as compared with three pin stator [27].

A common practice in the rub diagnosis was to investigate the rotor response and ignore the stator vibration features. The primary reason for such approach is the notion that the stator vibration data is a mixture of various vibrating components falling along the transfer path and the features related to rub would be difficult to extract. Therefore, most of the rotor model, intended to investigate rub, does not include the stator as a part of the system despite of the fact that the rotor and stator does have structural connectivity. In a structural model of HPTP (high pressure fuel turbopump), Childs [28] has accounted for the rotor-stator connectivity in the governing equation of motion to study the instability behaviour of various turbopump configurations. Choy et al. [29] have discussed the rotor-stator coupled model, whereas a simplified Jeffcott rotor model has been considered for the rub signature analysis. An aero-engine rotor system is modelled as rotor-bearing-support-stator coupled system and analysed for rub faults [30]. The rub related features based on casing vibration data and modulations of blade impact frequency by rotational frequency are proposed by Chen [31]. Further, with an improved aero engine rotor model, investigations are carried out on blade-casing rub with various rubbing types such as single point, multi-point, local part or complete cycle rubbing on the casing and on the rotor [32]. Fundamental frequencies, harmonics and combination frequencies were noted by Chen et al. [33] in the rotor response spectrum under rubbing between cylindrical shell and stator and the disc-stator contact. A clear identification is obtained for single and multiple rub impact.

The problem of rotor accessibility for direct vibration measurement has been encountered by many researchers and a comfortable location for sensor placement would be on bearing housing which itself has its own limitations. Therefore, the most preferable and easy location would be on the stator as it can be made accessible to all types of the vibration sensor such as proximity transducers, laser sensors, accelerometers etc. It would be of immense practical significance, if the measurement of stator vibrations at the casing location can give measurable and conclusive indication of the rub phenomenon.

In the present work, an FE based rotor-stator coupled system is modelled where the rotor vibration is transmitted to the stator through bearing housing and through direct contact. The rub interaction is defined using contact mechanics based Lagrange multiplier approach that ensures dynamic rotor-stator contact boundary and more realistic contact conditions in contrast to most of the earlier approaches. The stator thus responds through its own vibratory motion and the instantaneous clearance decides the nature of interaction. Periodic, quasi-periodic and