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Axial-transversal coupling in the nonlinear dynamics of a beam with an inclined roller

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ABSTRACT

The nonlinear oscillations of an Euler–Bernoulli beam hinged at one end and having a roller support sliding on a inclined line on the other end are investigated for the first time (to the best of the authors' knowledge). Free and forced dynamics are studied, and the backbone curve and frequency response curve are obtained by the multiple time scale method. These results allow us to detect analytically the main features of the nonlinear dynamics, in particular the hardening/softening dichotomy, and how it is influenced by the various beam parameters. Furthermore, the analytical results are compared with finite element simulations to check their reliability. The main goal of the paper is that of investigating the coupling between axial and transversal displacements, which appears yet in the linear regime (first order solution) due to the slope of the constraint. An element of novelty consists of showing the qualitative and quantitative changes that the coupling undergoes passing from small (linear regime) to moderate and large (nonlinear regime) displacements.

1. Introduction

The coupling between transversal (bending) and axial (longitudinal) displacements of a planar beam is an interesting dynamical phenomenon that, beside its theoretical interest, has also a practical significance since in some applications it can be useful to redirect oscillations, and the associated elastic and kinetic energy, from transversal to axial direction, in order to reduce the maximum transversal displacement, or viceversa. On the contrary, it may be dangerous if, for example, one needs only longitudinal oscillations and unwanted transversal oscillations appear due to coupling, similarly to a spillover effect [1]. It can occur both in the linear and in the nonlinear regime.

There are various forms of coupling. It may occur between purely axial and purely transversal modes, as a consequence of internal resonances [2], nonlinear [3] or other effects. Alternatively, it can be due to normal modes that have both axial and transversal displacements, which is the simplest case and which is the situation investigated in this work.

The coupling is due to different sources. In curved beams, it originates in the equilibrium equations, and thus entails linear normal modes having simultaneously axial and transversal components of the displacements. In this case coupling is observed yet in the linear regime [4], although of course it is a fortiori present in the nonlinear regime [5].

Coupling can occur also in initially straight beams. In the linear regime, it may appear when the cross-section is non homogenous and the center of mass is different from the center of flexural stiffness, like for example in composite layered beams [6] or in beams with a transverse through the thickness localized crack [7]. This effect has been extended to the 3D case, too, where the crack is seen to couple axial, transversal and torsional vibrations [8]. Also the coupling due to longitudinal crack has been studied [9].

When mass and stiffness centers coincide, like for example in beams with a symmetric cross-section, in the linear regime the axial and transversal dynamics are decoupled, and the coupling may be due to nonlinear terms, in particular to geometric nonlinearities [10]. The well-known "static" or "kinematic" condensation method [11], in which the axial inertia is neglected and the axial displacement is thus written as a function of the transversal one, is a remarkable example of this behavior.

In the nonlinear coupling case of straight beams, it may occur that second order axial displacement is observed as a consequence of the first order flexural oscillation [12], or it may happen that the first order decoupled axial and transversal oscillations couple to the second order [3]. The nonlinear coupling has been investigated also in the case of drillstrings, where it has been added to impact effects, both in planar [13] and 3D cases [14,15].

As noted by Ding and Chen [16], "under certain conditions, an axially moving beam may undergo transverse and longitudinal motions that are usually coupled if the geometrical nonlinearity has to be considered." In this case, in addition to the nonlinear geometric coupling, also nonlinear coupling appears in the inertial terms, these latter thus

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Fig. 1. The considered mechanical model. The slope is $\beta = \tan\alpha$.

further contributing to coupling [17]. The case of internal resonance has been investigated, too [18].

Like in axially moving beams, also in rotating beams the bending/longitudinal coupling is due to the inertia effects, in particular to Coriolis forces [19]. More generally, this is true also for flexible beams attached to a moving base [20].

In beam conveying fluids, the coupling between longitudinal and lateral vibrations is due to the interaction between the fluid and the beam, that introduces coupling terms in the governing equations [21]. The combined effects of the internal fluid flow and external wave and current loads has been investigated in [22].

The coupling between axial and transversal modes has been studied also in the case of strings [23], in particular referring to piano strings [27] and addressing the acoustic effects of this phenomenon, as well as in horizontal and inclined non-shallow cables [24,25].

In this paper a different mechanism is investigated, and the coupling due to boundary conditions is considered. More specifically, we study the coupling provided by an α-inclined, with respect to the axis of the rectilinear planar beam, roller support, as described in Fig. 1. This entails having coupling both in linear and nonlinear regimes. In the former, in particular, the normal modes have simultaneously axial and transversal components, while the nonlinear regime further enforces this behavior.

As a matter of fact, examples of straight beams with inclined support can be found in various engineering applications: an inclined ladder resting on a vertical wall with a roller on top, a beam suspended to inclined cables (as in hanging shelter suspended), a pedestrian walkway from a pier to a float, or from a boat to a pier, the access ramp of the roll on/roll of ferries, etc.

The kinematically exact equations of motion are considered in the framework of an Euler–Bernoulli direct beam model, and a linear constitutive model is considered (Section 2). The axial and transversal partial differential equation of motions are addressed by the multiple time scale method (Section 3), and no reduced order or Galerkin approximation are used [26]. The first order problem provides the linear behavior, while the nonlinear behavior is given by the second and third other problems, and is investigated in Section 4 by studying the backbone curve, in the free dynamics, and the frequency response curve, in the forced dynamics. To check the reliability of the proposed results, a comparison between the approximate analytical solution and a numerical solution obtained by the finite element method is reported in Section 5. The paper ends with some conclusions and suggestions for further developments (Section 6).

2. The mechanical model

We consider the initially rectilinear planar *slender* beam depicted in Fig. 1, referred to the inertial frame Z and X.

The measures of strain are

$$e = S' - 1, \qquad k = \frac{d\varphi}{dZ} = \varphi', \tag{1}$$

where the prime denotes derivative with respect to spatial variable Z (that ranges from 0 to the length of the beam L) and where:

- e is the elongation of the beam axis;
- S' is the axial stretch:

$$S' = \sqrt{(1+W')^2 + U'^2}; \tag{2}$$

- k the *mechanical* curvature [28];
- φ is the slope angle of the beam axis, and is given by

$$\cos\varphi = \frac{1+W'}{S'}, \quad \sin\varphi = \frac{U'}{S'}, \quad \tan\varphi = \frac{U'}{1+W'}; \tag{3}$$

- W and U are the displacements in the Z and X directions, respectively. To the first order, W and U are the axial and transversal displacements, respectively.

Since the beam is assumed to be slender, an *Euler–Bernoulli* model is considered, so that there is no shear strain and it is assumed that the cross-section remains perpendicular to the beam axis. It is worth to remark that we are considering an exact kinematic model.

The balance equations written in the deformed configuration are

$$\begin{aligned} &H_o' + q(Z,T) = \rho A \ddot{W} + c_W \dot{W}, \\ &V_e' + p(Z,T) = \rho A \ddot{U} + c_U \dot{U}, \\ &M' - V S' = 0, \end{aligned} \tag{4}$$

where dot denotes derivative with respect to the time T, and where:

- H_o and V_e are the horizontal (in the Z-direction) and vertical (in the X-direction) internal forces, respectively:

$$H_o = N\cos\varphi + V\sin\varphi, \qquad V_e = N\sin\varphi - V\cos\varphi; \tag{5}$$

- N, V and M are the axial force, shear force and bending moment, respectively;
- ρA is the mass per unit length in the reference configuration, which is assumed to be constant;
- c_W and c_U are the damping coefficients;
- $q(Z, T)$ and $p(Z, T)$ are the distributed loads in the Z and X directions, respectively;
- the rotational inertia has been neglected since the beam is slender.

The beam is assumed to be homogenous and linearly elastic, so that the constitutive laws are

$$N = EAe, \qquad M = EJk. \tag{6}$$

EA and EJ are the axial and bending stiffnesses, respectively, and are the unique constitutive constant parameters of the model.

Inserting (2) in $(1)_1$ and $(3)_3$ in $(1)_2$ gives

$$e = \sqrt{(1+W')^2 + U'^2} - 1, \qquad k = \left[\arctan\left(\frac{U'}{1+W'}\right)\right]', \tag{7}$$

so that, by (6) and $(4)_3$ we get

$$\begin{aligned} N &= EA\left(\sqrt{(1+W')^2 + U'^2} - 1\right), \\ M &= EJ\left[\arctan\left(\frac{U'}{1+W'}\right)\right]', \\ V &= \frac{M'}{S'} = EJ\frac{\left[\arctan\left(\frac{U'}{1+W'}\right)\right]''}{\sqrt{(1+W')^2 + U'^2}}. \end{aligned} \tag{8}$$

Inserting these expressions in (5) (and using $(3)_1$ and $(3)_2$) yields

$$\begin{aligned} H_o &= EA\left(\sqrt{(1+W')^2 + U'^2} - 1\right)\frac{1+W'}{\sqrt{(1+W')^2 + U'^2}} \\ &\quad + EJ\frac{\left[\arctan\left(\frac{U'}{1+W'}\right)\right]''}{\sqrt{(1+W')^2 + U'^2}}\frac{U'}{\sqrt{(1+W')^2 + U'^2}}, \\ V_e &= EA\left(\sqrt{(1+W')^2 + U'^2} - 1\right)\frac{U'}{\sqrt{(1+W')^2 + U'^2}} \\ &\quad - EJ\frac{\left[\arctan\left(\frac{U'}{1+W'}\right)\right]''}{\sqrt{(1+W')^2 + U'^2}}\frac{1+W'}{\sqrt{(1+W')^2 + U'^2}}. \end{aligned} \tag{9}$$

Finally, inserting (9) in the balance equation $(4)_1$ and $(4)_2$ provides the following two equations in the two unknowns $W(Z, T)$ and $U(Z, T)$: