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A history-dependent cohesive zone model in elastic and visco-elastic materials under constant and variable loading

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ABSTRACT

A history-dependent cohesive zone model approach is used to study the crack behaviour in elastic and visco-elasto materials. The cohesive (yield) stress at the cohesive zone points is related to the nonlinear normalised equivalent stress functional over the stress history at these points, and is expressed in the form of an Abel-type (fractional) integral. We analyse the cohesive zone length evolution in time and the crack tip opening during the stationary crack stage as well as during the propagating crack stage. We consider the external load increasing linearly with time and compare the solution with the case of the constant load. We obtain the solution numerically and analyse the influence of the viscoelasticity by comparing with the case of purely elastic behaviour of the bulk of the material.

1. Introduction

An important assumption needed in order to use linear elastic fracture mechanics is that the inelastic region at the crack tip must be negligible in comparison to the size of the whole crack itself. However, in many situations this assumption does not hold. In such cases, elasto-plastic fracture mechanics is considered instead, which particularly includes cohesive zone models. They allow modelling of short and long crack growth as well as crack nucleation and initiation. In such models, it is assumed that there exists a cohesive zone, CZ, which is the area between two separating but still sufficiently close surfaces ahead of the crack tip, see the shaded region in Fig. 1.

At this cohesive zone, cohesive tractions pull the cohesive faces together. The external loads applied to a body cause the crack to propagate and could also cause the crack faces to move further away from each other. By implementing a cohesive zone model, we would like to find out the size of this cohesive zones, and more importantly, when the crack will start to propagate and how fast will it propagate thereafter. When the crack propagates, the cohesive forces vanish at the points where the cohesive zone opening reaches a critical value and these points become the crack surface points, while the new material points, where the history-dependent normalised equivalent stress reaches a critical value, join the cohesive zone. So, the CZ is practically attached to the crack tip ahead of the crack and moves with the crack, keeping the normalised equivalent stress finite in the body.

There have been many cohesive zone models (employing different traction-separation laws) introduced in the literature. The simplest model, introduced by Leonov–Panasyuk–Dugdale (LPD) (1959–1960), see [3,8], is when the cohesive stress, that can be associated with the yield stress, σ_y, is constant, while the bulk of the material is elastic. This model is one of the most popular cohesive zone models and many modifications of this model have been made and widely used in nonlinear fracture mechanics.

The 3 main compounds of a cohesive zone model are:

- the constitutive equations in the bulk of the material;
- the constitutive equations in the cohesive zone;
- the criterion for the cohesive zone to break, i.e., the crack to propagate.

The model presented in this paper is an extension of the LPD model to linear visco-elastic behaviour of the bulk of materials with non-linear history-dependent constitutive equations in the cohesive zone. Our aim is to find the time-evolution of the CZ before the crack starts propagating, the delay time, after which the crack will start to propagate, and model further the time-evolution of the crack and the CZ. In all these stages we assume the quasi-static evolution of crack and cohesive zones, i.e., the dynamic effects are not considered. The case of a constant external load was studied in [6], with some preliminary results published in [7]. The focus of this paper is to show that the suggested model and methods for solving the corresponding crack problems are equally applicable to the external load variable in time.

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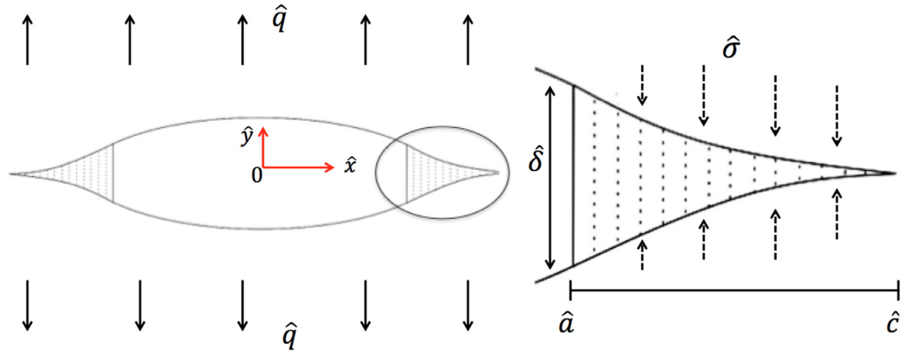

Fig. 1. Cohesive zone.

2. Problem formulation

Let the problem geometry be as in Fig. 1, i.e., the crack occupies the interval $[-\hat{a}(\hat{t}), \hat{a}(\hat{t})]$ and the cohesive zone occupies the intervals $[-\hat{c}(\hat{t}), -\hat{a}(\hat{t})]$ and $[\hat{a}(\hat{t}), \hat{c}(\hat{t})]$ in an infinite linearly elastic or viscoelastic body under plane strain or plain stress conditions. The body is loaded at infinity by traction $\hat{q}(\hat{t})$ in the direction normal to the crack, applied at the time $\hat{t} = 0$ and is constant in the coordinate $\hat{x}$. The crack is traction-free, i.e., $\hat{\sigma}_{\hat{y}\hat{y}}(\hat{x}, 0, \hat{t}) = 0$ for $|\hat{x}| < \hat{a}(\hat{t})$. To simplify the notations, we denote the normal stress ahead of the crack as $\hat{\sigma}$, i.e., $\hat{\sigma}(\hat{x}, \hat{t}) = \hat{\sigma}_{\hat{y}\hat{y}}(\hat{x}, 0, \hat{t})$. The initial CZ tip coordinate and crack tip coordinate are prescribed, $\hat{c}(0) = \hat{a}(0) = \hat{a}_0$, while the functions $\hat{c}(\hat{t})$ and $\hat{a}(\hat{t})$ for time $\hat{t} > 0$ are to be found.

We will now formulate and normalise the principal equations while considering two cases: a constant external load $\hat{q}$ and an external load $\hat{q}(t)$ varying in time.

2.1. Natural form of the cohesive zone condition

First, as in [6], we will replace the LPD cohesive zone stress condition, $\sigma = \sigma_y$, with the history-dependent condition

$$\underline{\Lambda}(\hat{\sigma}; \hat{t}) = 1, \tag{1}$$

where

$$\underline{\Lambda}(\hat{\sigma}; \hat{t}) = \left(\frac{\beta}{b\sigma_0^\beta} \int_0^{\hat{t}} |\hat{\sigma}(\hat{\tau})|^\beta (\hat{t} - \hat{\tau})^{\frac{\beta}{b}-1} d\hat{\tau} \right)^{\frac{1}{\beta}} \tag{2}$$

is the normalised history-dependent equivalent stress, $|\hat{\sigma}|$ is the maximum of the principal stresses, and $\hat{t}$ denotes time.

The parameters σ_0 and b are material constants in the assumed power-type relation

$$\hat{t}_\infty(\hat{\sigma}) = \left(\frac{\hat{\sigma}}{\sigma_0} \right)^{-b} \tag{3}$$

between the physical rupture time $\hat{t}_\infty$ and the constant uniaxial tensile stress $\hat{\sigma}$ applied to a sample without cracks. These parameters can be obtained by fitting the creep durability experimental data on macro-samples. Here, b is dimensionless, $\hat{t}_\infty$ has units of time, e.g., seconds, s, and if the stress $\hat{\sigma}$ is in Pascals, Pa, then $\hat{\sigma}_0$ has units Pa·s$^{1/b}$. For many structural materials the parameter b is in the range between 5 and 20, cf. e.g. [10] and references therein. The dimensionless parameter β is a material constant in the nonlinear accumulation rule for durability under variable load, see [10]. Further details of this model can be found in [4,6].

Note that relations (1)–(2) were implemented in [5,11] to solve a similar crack propagation problem without a cohesive zone; i.e. it was assumed that when condition (1) is reached at a point, the crack spreads to this point. However, such local approach appeared to be inapplicable for $b \geq 2$. In this paper, a cohesive zone approach is developed instead, in order to cover the larger range of b values relevant to structural materials. In the CZ approach, when condition (1) is reached at a point, the cohesive zone spreads to this point.

As proved in [6, Section 3.2], the CZ model is applicable only if material parameters, b and β, of the history dependent yield condition, are such that $b > 0$, $0 < \beta < b$. This implies that the CZ model is not applicable for the Robinson-type history-dependent yield condition, based on the power-type durability diagram, for which $\beta = b$.

The cohesive zone condition (1)–(2) at a point $\hat{x}$ on the cohesive zone can be rewritten as

$$\int_{\hat{t}_c(\hat{x})}^{\hat{t}} \hat{\sigma}^\beta(\hat{x}, \hat{\tau})(\hat{t} - \hat{\tau})^{\frac{\beta}{b}-1} d\hat{\tau} = \frac{b\sigma_0^\beta}{\beta} - \int_0^{\hat{t}_c(\hat{x})} \hat{\sigma}^\beta(\hat{x}, \hat{\tau})(\hat{t} - \hat{\tau})^{\frac{\beta}{b}-1} d\hat{\tau}, \tag{4}$$

for $\hat{t} \geq \hat{t}_c(\hat{x})$ and $\hat{a}(\hat{t}) \leq |\hat{x}| \leq \hat{c}(\hat{t})$. Here, $\hat{t}_c(\hat{x})$ denotes the time when the cohesive zone spreads to the point $\hat{x}$. Eq. (4) is an inhomogeneous nonlinear Volterra integral equation of the Abel type (fractional integral equation) with unknown function $\hat{\sigma}^\beta(\hat{x}, \hat{t})$ for $\hat{t} \geq \hat{t}_c(\hat{x})$.

2.2. Normalised form of the cohesive zone condition

To simplify condition (4), and other equations further on, we normalise the variables, which will make them dimensionless and reduce the number of significant parameters. This also make the comparison of the results for constant and variable load cases more illustrative.

For the *constant loading* case, $\hat{q}(\hat{t}) = \hat{q}_0$ is independent of time, and using Eq. (3) we denote

$$\hat{t}_\infty = \hat{t}_\infty(\hat{q}_0) = \left(\frac{\hat{q}_0}{\sigma_0} \right)^{-b}.$$

Then we can introduce the normalised time, point coordinate, crack tip coordinate, cohesive zone tip coordinate, and stress as follows,

$$t = \frac{\hat{t}}{\hat{t}_\infty}, \quad x = \frac{\hat{x}}{\hat{a}_0}, \quad a(t) = \frac{\hat{a}(t\,\hat{t}_\infty)}{\hat{a}_0}, \quad c(t) = \frac{\hat{c}(t\,\hat{t}_\infty)}{\hat{a}_0}, \quad \sigma(x, t) = \frac{\hat{\sigma}(x\,\hat{a}_0, t\,\hat{t}_\infty)}{\hat{q}_0}, \tag{5}$$

and the normalised external load becomes $q(t) = \hat{q}(\hat{t})/\hat{q}_0 = 1$.

The situation is a bit more complicated when the load is time-dependent, particularly, when it is given by a linear function, i.e., $\hat{q}(\hat{t}) = \dot{q}\,\hat{t}$, with $\dot{q} = const$. Let us denote by $t_{\bullet\infty}$ the corresponding rupture time, i.e., the time when the cohesive zone spreads over the infinite plane without crack, under the time-variable load. To obtain $\hat{t}_{\bullet\infty}$, we consider condition (1)–(2) for *delete* $\hat{\sigma}(\hat{\tau}) = \hat{q}(\hat{\tau}) = \dot{q}\,\hat{\tau}$, which reduces to

$$\dot{q}^\beta \int_0^{\hat{t}} \hat{\tau}^\beta (\hat{t} - \hat{\tau})^{\frac{\beta}{b}-1} d\hat{\tau} = \frac{b\sigma_0^\beta}{\beta}. \tag{6}$$

Expressing the integral in (6) in terms of the Beta-function and solving the equation for $\hat{t} = \hat{t}_{\bullet\infty}$ gives

$$\hat{t}_{\bullet\infty} = \left(\frac{\sigma_0}{\dot{q}\alpha} \right)^{\frac{b}{1+b}}, \tag{7}$$