International Journal of Mechanical Sciences 142–143 (2018) 66–73

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

An empirical non-proportional cyclic plasticity approach under multiaxial low-cycle fatigue loading

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ARTICLE INFO

Keywords:
Non-proportional hardening
Multiaxial loading
Cross hardening
Incremental plasticity
Consistency condition

ABSTRACT

The non-proportional hardening is revealed to be a main reason to influence the fatigue life of materials which are sustained to multiaxial low-cycle fatigue loading. Microstructurally speaking, increase of stacking faults to the grain size, formation of micro-twins in deformation bands and/or nucleation of martensite particles under non-proportional loading can introduce the strain hardening effect. Nevertheless to quantify the correlation between the microstructural phenomena and the macrostructural characteristic of material is not yet a trivial task. After quantitative analysis of the correlation between microstructural development and periodic deformation behaviour, an empirical non-linear hardening approach based on Benallal's *NP* factor estimation, which reflects the relationship between microstructural condition and internal stress responses is presented in this paper. Our approach is verified to be effective by experimental observations under different loading paths.

1. Instruction

Multiaxial fatigue load histories can be proportional or non-proportional. The proportional loadings can cause isotropic strain-hardening effects. On the other hand, The NP loadings may produce an additional effect called NP hardening caused by the fluctuation of principal stress/strain directions for certain materials. [1–5]. In comparison with proportional loading, such additional hardening effect can appear, for example, in stainless steel under *NP* loading, but this cannot happen in some aluminum materials. Under *NP* histories, it is not surprising that more stacking faults can be generated for materials with lower *SFE* such as SS316L than for materials with higher *SFE* such as aluminum 7075 possessing the same plastic straining. Therefore, more difficulties for cross-slip across such large stacking faults produce significant increase of stress response [6–9]. Itoh proposed an empirical *NP* factor to correlate the *NP* strain range $\Delta\varepsilon_{NP}$ for predicting the *NP* low cycle fatigue lives [10],

$$\Delta\varepsilon_{NP} = \left(1 + \alpha_{NP} \cdot F_{NP}\right)\Delta\varepsilon_I \tag{1}$$

where $\Delta\varepsilon_I$ was the maximum principal strain range under *NP* straining, α was the constant material parameter and F_{NP} was the *NP* factor which showed the severity of *NP* loading. α became larger for lower *SFE* materials based on experimental results as shown in Table 1 [11].

Although there are several definitions of the origin of *NP* hardening, it is widely accepted that the additional hardening observed under *NP* multiaxial fatigue loading is mainly ascribed to large stacking faults. However, the formation of twins or labyrinths in deformation bands and even the nucleation of martensite particles during large plastic deformation cannot be ignored either in the microstructural estimation of the additional hardening for some materials [12–14]. Kida et al. described the microstructure of SS304 after a series of *NP* loading paths, detailed in [15]. This experimental observation leads us to conclude that the additional hardening has a close connection with dislocation structures which have different forms produced by different loading paths. First of all, stacking faults have been observed in all tests, no matter under proportional or *NP* conditions, but the proportion of SF alters according to loading histories. For example, the number of SF in X-crossed path is greater than that in cruciform path and when the same strain range is applied larger stress response is produced in former case. Furthermore, more complex condition appears because of the observation of martensite particles, which can largely increase material stress response, such as in square path case. Martensite transformation was also confirmed in stainless steel material with high plastic deformation [16] from the metastable austenitic phase (γ) to ε (HCP) and α' (BCC) martensitic phases. According to [17], α' martensite nucleates at the intersections of stacking faults, shear bands, and then grows and thickens to its stable form at about 5–7 nm. Besides, Mazánová et al. observed that higher Ni content in SS316L used in earlier investigations and the drop in Ni content in steels was related to the unstable cyclic hardening behavior [18]. Hence, from the mechanical point of view, the *NP* hardening may be qualitatively related to stacking faults, dislocation structures and/or martensite particles when accumulated plastic strain is high enough. Some studies on the correlation between microstructure of material and

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https://doi.org/10.1016/j.ijmecsci.2018.04.031
Received 7 June 2017; Received in revised form 9 April 2018; Accepted 18 April 2018

Nomenclature

$d\vec{\beta}$ = backstress increment vector
$\vec{e}$ = deviatoric strain vector
$\vec{s}$ = deviatoric stress vector
$d\vec{p}$ = equivalent plastic strain increment vector
$\vec{n}$ = unit normal vector
$d\overrightarrow{\varepsilon_{\mathrm{pl}}}$ = plastic strain increment vector
$\vec{\varepsilon}$ = strain vector
$\vec{\upsilon}$ = unit translation direction vector
$[P_T]$ = fourth order polarization tensor proposed by Tanaka
$< >$ = MacCauley bracket
$\vec{\sigma}$ = stress vector
A = Benallal's non-proportionality parameter
a_{x1}, a_{y1}, a_{z1}, a_{x2}, a_{y2}, a_{z2} = coordinate transformations coefficients
α_{ij} = angle between normal stress and deviatoric plastic strain
E = elasticity modulus
F_{NP} = non-proportional factor
G = shear modulus
H_c = Ramberg–Osgood uniaxial cyclic hardening coefficient
h_c = Ramberg-Osgood's uniaxial cyclic hardening exponent
hr_c = uniaxial isotropic strain hardening rate
hr_{NP} = non-proportional hardening rate
k_s = scaling factor
NP = non-proportional
p = accumulated plastic strain
P = generalized plastic modulus
p_i = generalized plastic modulus coefficients
s_1, s_2 = user-defined deviatoric stresses on the deviatoric plane
s_3, s_4, s_5 = user-defined deviatoric stresses proportional to the shear stresses τ_{xy}, τ_{xz} and τ_{yz}.
SFE = stacking faults energy
S_y = monotonic yield stress
S_{yc} = cyclic yield stress
α_{NP} = material parameter related to additional hardening
Δr_i = i^{th} yield surface saturation values
$\Delta\varepsilon_I$ = maximum principal strain range
$\Delta\varepsilon_{NP}$ = non-proportional strain range
ε_h = hydrostatic strain
θ_{ij} = angle between stress and plastic strain proposed by Benallal
ξ = additional weight parameter
σ_h = hydrostatic stress
υ = Poisson's ratio
φ_s = rotation angle
χ_i, m_i, γ_i, δ_i = adjustable ratcheting parameters

Superscripts
eng = engineering
T = transpose
' = in 5D deviatoric spaces

Subscripts
el = elastic
pl = plastic

additional hardening effect have been reported in [19–22] but these issues still need long term endeavours.

On the other hand, metallic material failure takes place usually on specific plane under proportional loading due to the fixed orientation of the principal stress. However, as *NP* periodic histories induce the principal stress directions that vary along the time, the fatigue damage in different planes need to be evaluated to find out the crack initiation plane whose damage is maximized according to some criterions suitable for the material. The macromechanical parameters e.g. normal and shear stress directions, which depend on the actual shape of the loading path, have been qualitatively proved to be associated with microstructure effect of material. The *NP* factor F_{NP}, ratio of the shear to normal amplitude, reflects the relation of microstructures of material and loading path applied. Accordingly, the macroscopic response of the polycrystal is physically linked to the evolution of dislocation microstructure during plastic deformation. Benallal and Marquis [23] proposed a *NP* factor estimation that took into account the angle between the actual stress and the plastic strain. Tanaka [24] proposed a robust *NP* parameter taking the internal dislocation structure formed by the loading process as a fourth order polarization tensor. The physically-based models accounting for microstructural dislocations during complex deformation states have been confirmed to be associated with orthogonal loading-path changes. In subsequent orthogonal paths, those dislocations act as latent resistance to yielding and the increase hardening rate [25].

In this paper, by taking into account of the angle between the stress and the plastic strain increment, an empirical constitutive relation estimation incorporated with incremental plasticity method is presented. The approach proposed is validated through comparison of the experimental and predicted results of SS316L tubular specimens under different *NP* loading path histories.

2. Deviatoric spaces

Fundamentally, the additional hardening or the fatigue damage results from the change of material structural at the microscopic scale. But to quantify microscopic and macroscopic parameters is still far away from being settled. Actually, it is reasonable to believe that the macroscopic parameters governing additional hardening have an inherent relationship with the internal dislocation structure parameters as proposed by Tanaka or the macroscopic stress and strain directions as proposed by Benallal, which are relatively convenient to be implemented in deviatoric spaces. Now, before embarking on the investigation into the

Table 1
NP hardening factors for several materials [11].

Material	*αNP*
316 stainless	1
304 stainless	0.5–1.0
316 stainless (550 °C)	0.37
OFHC copper	0.3
1045 steel	0.3
304 stainless (650 °C)	0.3
Inconel 718	0.2
Al 6061-T6	0.2
42 CrMo steel	0.15
1% Cr Mo-V steel	0.14
En15R	0.14
Al 7075	0.0
Al 1100	0.0