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# Parametric resonance analysis of rectangular plates subjected to moving inertial loads via IHB method

Mostafa Pirmoradian[a,*], Ehsan Torkan[b], Hossein Karimpour[c]

[a] Department of Mechanical Engineering, Khomeinishahr Branch, Islamic Azad University, Khomeinishahr, Isfahan 84175-119, Iran
[b] Young Researchers and Elite Club, Khomeinishahr Branch, Islamic Azad University, Khomeinishahr, Isfahan 84175-119, Iran
[c] Department of Mechanical Engineering, Faculty of Engineering, University of Isfahan, Isfahan 81746-73441, Iran

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ABSTRACT

This paper deals with the induced instability due to parametric resonance of rectangular plates traversed by inertial loads and lying on elastic foundations. The extended Hamilton's principle is employed to derive the partial differential equation associated with the transverse motion of the plate. Subsequently, this equation is transformed into a set of ordinary differential equations by the Galerkin method. Including vertical, centripetal and Coriolis acceleration terms related to the moving mass transition in the analysis leads to governing equations with time-varying mass, damping and stiffness coefficients. Particularly, the intermittent passage of masses along rectilinear paths, or the motion of an individual mass along an orbiting path, permits to subcategorize the problem as a parametrically excited system with periodic coefficients. By applying the incremental harmonic balance (IHB) method, the stability of the induced plate vibrations is investigated, revealing an emersion of instability tongues in the parameters plane. Semi-analytical results are provided for various boundary conditions of the plate which got verified through direct numerical simulations and other results reported in the literature.

## 1. Introduction

The study of the dynamic behavior of elastic structures under moving loads is of great significance in many fields of engineering such as transporting systems, overhead cranes, machining processes such as milling and turning, weapon firing barrels, etc., attracting many researchers' interest. Fryba's monograph [1] contains many contents on the fundamental postulations of moving load problems and their analytical solutions. Also, Ouyang [2] performed an extensive literature review on the moving load problem and associated engineering applications.

The topic of elastic media influenced by moving objects embraces comprehensive research works devoted to cables, beams and plates. Namely, Stahl [3] presented techniques to study the dynamic response of circular disks excited by moving loads. Shadnam et al. [4] studied the dynamic behavior of nonlinear plates under relatively large moving masses as a Duffing equation with time-varying coefficients. In another work, Shadnam et al. [5] examined periodicity in the response of nonlinear plates excited by moving masses through applying the Banach's fixed-point theorem. Kononov and Borst [6] studied the stability of transverse vibrations due to a mass moving uniformly over four different elastic systems including Euler–Bernoulli and Timoshenko beams and also Kirchhoff and Mindlin plates. Gbadeyan and Dada [7] examined the elastodynamic response of a rectangular Mindlin plate subjected to a uniformly distributed series of moving masses. Wu [8] performed the dynamic analysis of an inclined plate subjected to inertial loads, remarking the influence of Coriolis and centrifugal forces on the vertical deflections at higher speeds. Nikkhoo and Rofooei [9] evaluated the influence of all out-of-plane translational components of the mass acceleration on the dynamic behavior of the plate-mass system. The dynamic response of geometrically nonlinear thin rectangular plates under moving mass excitation was evaluated by Enshaeian and Rofooei [10] and Rofooei et al. [11]. Esen [12] presented a new finite element method to study dynamics of thin plates under traversing inertial loads. A similar method was used in [13] for moving loads with variable velocities. Amiri et al. [14] carried out a study on vibrations of a Mindlin elastic plate with different boundary conditions and multiple loading paths via the eigenfunction expansion method. Nikkhoo et al. [15] investigated the resonance conditions for a single-span thin rectangular plate traversed by a series of masses. Song et al. [16] presented a comprehensive method to predict dynamic behavior of flat plates with arbitrary boundary conditions subjected to moving masses based on Kirchhoff's plate theory. Foyouzat et al. [17] studied the dynamics of a viscoelastic plate resting on a viscoelastic Winkler foundation under the transition of moving masses using the Laplace transform. Wang and Zu [18,19] investigated nonlinear dynamic response of longitudinally travelling functionally graded material plates using the harmonic balance method.

* Corresponding author.
*E-mail address:* pirmoradian@iaukhsh.ac.ir (M. Pirmoradian).

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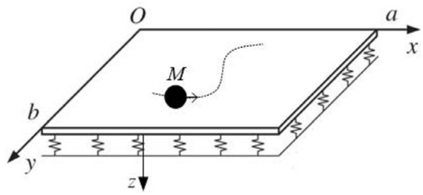


**Fig. 1.** Moving mass $M$ travelling along an arbitrary path.

Early studies on load-structure interaction were contrived based on neglecting the inertia of the moving objects, reducing the problem to a moving force one. Involving the inertial effects of the moving loads into the dynamical formulation leads to a more realistic and comprehensive treatment of the problem. By considering this model, time-dependent terms will appear in the governing equations of motion due to the prescribed motion of the mass along the structure, which may induce parametric excitation besides other external terms.

Parametric excitation may occur in a variety of mechanical, electrical, optical, and other physical systems when some characteristic properties of the system change periodically over time [20–24]. Most attentions concentrate on a unique and very important type of resonance called parametric resonance; in case the frequency of the periodic coefficients is nearly twice one of the natural frequencies of the system, causing the structure to steadily experience a net absorption of energy, which consequently leads toward instability. Nowadays, the literature is abundant concerning the field of parametrically excited systems. For example, one can cite the fluttering instability of structural elements such as plates and shells in mechanical and aerospace applications [25,26], the parametric resonance of axially moving plates [27], the dynamic buckling of axially excited cantilever beams [28] or the inverted pendulum balancing by base oscillation [20,29], to enumerate a few.

The moving mass problem perfectly illustrates the situation where parametric excitation can result in resonance. As alluded, the periodic nature of such systems is due to the intermittent passage of moving masses over the base structure [30–37]. A first attempt to reveal this type of instability on an Euler–Bernoulli beam carrying moving masses was addressed by Nelson and Conover [30] using the Floquet theory. Katz el. al. [31] investigated the dynamic stability of a simply supported beam subjected to a deflection-dependent moving load caused by cutting forces in machining operations. They concluded that a specific repetitive periodic trend was required to reach instability. Rao [32] studied the parametric as well as internal and external resonances of an Euler–Bernoulli beam under moving load and moving mass excitations. All acceleration terms related to the moving mass motion were included and their significance was highlighted, emphasizing on the role of inertial terms as sources of parametric and internal resonances. Makertich [33] developed the Timoshenko beam theory to study the dynamic stability of a beam excited by a sequence of moving masses. Pirmoradian et al. [34,35] utilized the IHB method to study the dynamic stability of simply supported Timoshenko and Euler–Bernoulli beams under the periodic loading of successive moving masses. They obtained some curves in the stable region of the system's parameter plane which present resonant behaviors. Karimpour et al. [36] addressed the coexistence phenomenon in a beam-moving mass system and mentioned its importance in engineering design.

Most related studies on dynamic stability of elastic media interacting with moving masses focused on beams as the base structure and to the best of the authors' knowledge, no study has been reported on the parametric resonance arising from plate-moving mass reciprocal action. Plate is considered as a two-dimensional structure and its related mathematical model is more complex in comparison with one-dimensional structures such as beams and cables, but also involves more versatile types of boundary conditions which is a key factor in determining stability verges.

In this paper, the instability phenomenon due to parametric resonance of a plate under the influence of successive moving masses is studied analytically. Disregarding the possible mechanism of instability via parametric excitation, and just dealing with time or frequency analyses may lead to fail in detecting resonance conditions. The IHB method is applied to find the boundary curves of the instability regions in the parameter plane for a diversity of plate boundary conditions. Three different loading paths are considered for the motion of moving masses over the plate's surface, including rectilinear, diagonal, and orbiting paths. For the rectilinear and diagonal paths, it is assumed that the sequence of moving masses is parallel to one edge of the plate or along its diameter, which is in accordance to the common loading of slab-type bridges, aircraft landing carriers and road pavements. As it concerns the orbiting path, it is supposed to be of considerable interest in the analysis of rotating machineries, guided circular saws commonly used in wood processing industries, hard disk drives, vehicles discs and drum brakes, etc. Finally, the time history of the plate's midpoint vibration is obtained via numerical integration to verify the semi-analytical results.

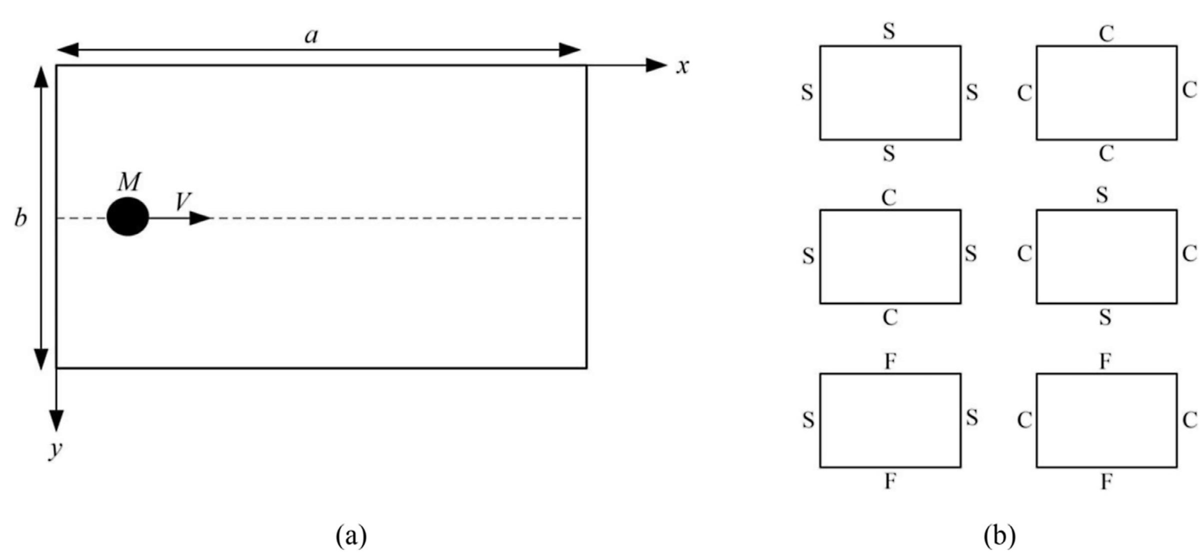


**Fig. 2.** (a) Moving mass $M$ travelling along a rectilinear path, (b) Plate boundary conditions; S, C and F stand for simple, clamped and free edges, respectively.