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Non-rotating beams isospectral to rotating Rayleigh beams

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ABSTRACT

In this paper, we establish spectral equivalence between certain non-uniform rotating Rayleigh beams and a given uniform Rayleigh beam, and also between non-uniform non-rotating Rayleigh beams and a given uniform rotating Rayleigh beam. Spectrally equivalent systems are those that have the same spectra, i.e., the same free vibration natural frequencies for a given boundary condition. We use a transformation to convert the non-dimensional Rayleigh beam equations from the (x, W) frame of reference to a hypothetical (z, U) frame of reference. If the material and geometric properties of the beam are specific chosen functions of the two introduced auxiliary variables, then the transformation will be achieved. If the coefficients of the transformed equation match with the required equation, then the equation, on which the transformation is applied, is said to be isospectral to the required one. The mode shapes for these particular cases also match due to the nature of assumption on one of the introduced auxiliary variables. Frequency and mode shape equivalence of the beams is confirmed by the finite element method (FEM). For the considered cases, examples of beams having a rectangular cross-section are presented to show the application of our analysis.

1. Introduction

Rotating elastic beams with uniform and varying cross-sections are used as structural elements in a wide range of engineering fields. They serve as useful mathematical models to simulate vibration of helicopter blades, long flexible rotating space booms, wind turbines, aircraft propellers, turbo machinery blades etc. Hence, the study of free vibration of such beams is of great importance for understanding mechanical behaviour and structural reliability of these components.

Long and slender beams can be accurately modeled using the Euler–Bernoulli (EB) beam theory; whereas, for short and thick beams, and for accurate frequency prediction of the higher modes of vibration, the Bresse–Timoshenko (BT) beam theory is more widely used. Free vibration studies of rotating Rayleigh [1,2] and Timoshenko beams [3–7] are available in the literature. Lord Rayleigh [8] developed a relatively "simpler theory" before the Timoshenko beam theory came into existence which includes the rotary inertia effect but does not take into account the shear deformation [1,9–11]. The Rayleigh beam theory predicts the natural frequencies and mode shapes more accurately than the Euler–Bernoulli beam theory while avoiding the mathematical complexities of the Timoshenko beam theory.

Inverse problems can be stated as finding the cause of a given effect or finding a law of evolution given the cause and effect [12–14]. Included among such problems in the context of vibration eigenvalue problems are many questions like- can we find beams which have given eigenvalues and eigenvectors? What spectral data is necessary and sufficient to ensure that the system, if it exists at all, is unique? Finding material and geometric properties from known modal parameters and reconstruction of a beam from its spectral data [15–17] is one such problem. Multiple beams can have same spectra for a given boundary condition. The existence of systems that have the same frequencies for a given boundary condition but have different material and geometric properties is of great interest in mechanics. An important subclass of inverse problems is finding isospectral systems which involve finding beams having the same spectra as that of a known beam.

Isospectral Euler–Bernoulli beams with continuous density and rigidity functions were analyzed by Gottlieb in [18]. Gladwell and Morassi [19,20] studied discrete isospectral systems and described ways to form inline spring-mass systems isospectral to a given one. Borg [21] studied vibrating strings with continuous coefficients. Subramanian and Raman [22] generalized the transformation to obtain isospectral systems for all taper powers. Ghanbari [23] found twelve classes of isospectral beams by factoring the fourth-order beam operator into two second-order differential operators for four different boundary conditions. Gladwell and Morassi [24] considered a specific class of beams where the product of stiffness and mass per unit length is constant. Boundary conditions being any combination of pinned and sliding, they obtained a closed form expression for beams isospectral to a given beam. The special class of beams was equivalent to a string and Darboux lemma was

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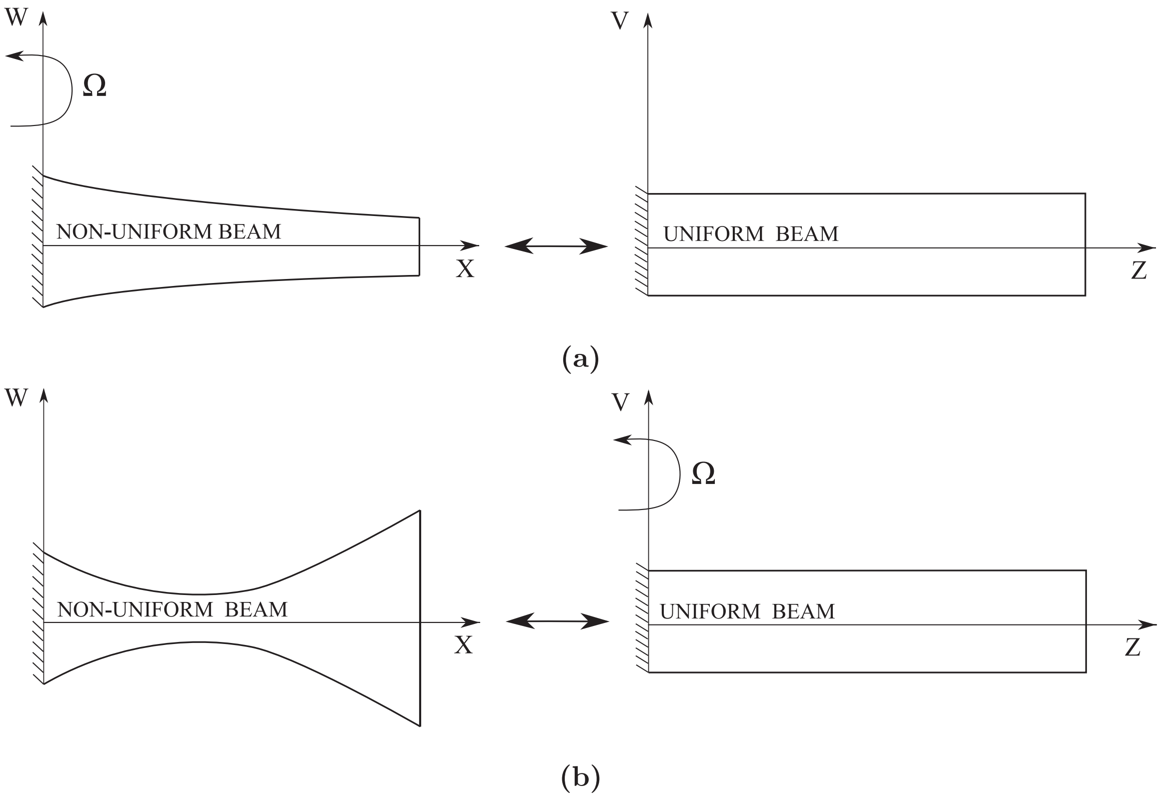

Fig. 1. Schematic of (a) non-uniform rotating beam isospectral to a uniform non-rotating beam and (b) non-uniform non-rotating beam isospectral to a uniform rotating beam.

used to reduce the string equation to Sturm-Liouville canonical form. In [25,26], a procedure for obtaining real densities of circular membranes that are isospectral to a given uniform circular membrane under fixed and free boundary conditions was introduced by Gottlieb. Seven different classes of beams were found to be analytically solvable and isospectral to a homogeneous beam and corrections to the transformation used by Barcilon were made in [27]. In [25] it was shown that membranes isospectral to radial density membranes no longer possess radial symmetry.

Non-uniform and rotating beams are extensively studied and are available in the literature. Rajasekaran and Khaniki [28] studied the mechanical behaviour of non-uniform small scale beams in the framework of nonlocal strain gradient theory. Khaniki and Hosseini-Hashemi [29] studied buckling behaviour of tapered small-scale beams in the framework of nonlocal strain gradient theory. Rajasekaran and Bakhshi Khaniki [30] presents mechanical behaviours of nonhomogenous nonuniform size-dependent beams using nonlocal strain gradient theory. They used finite element method in conjunction with numerical integration, Gaussian quadrature method, and Wilson's Lagrangian multiplier to solve problems considered. Khaniki [31] provides an investigation on transverse vibrational behaviour of rotating cantilever size-dependent beams using the framework of Eringen's two-phase local/nonlocal model and solving it using a modified generalized differential quadrature method.

Kambampati et al. in [32] found non-uniform rotating beams isospectral to a given uniform rotating beam. In another work [33,34], they found non-rotating beams isospectral to rotating uniform beams and rotating beams isospectral to axially loaded non-rotating uniform beams. Kambampati and Ganguli in [35,36] found non-uniform beams and stiff springs isospectral to axially loaded uniform beams and piano strings, and non-rotating beams isospectral to tapered rotating beams. In their study, they used Barcilon–Gottlieb transformation to convert the fourth-order governing equation of one type of beam to the required one. Then they validated the results using FEM and provided examples of isospectral rectangular cross-section beams as the application of their analysis. A practical significance of this work was that tests to determine rotating or axially loaded beams could be replaced by easier tests on non-rotating beams with different mass and stiffness distributions. Also, Bhat and Ganguli in [37] have arrived at classes of non-uniform Rayleigh beams which are spectrally equivalent to uniform Rayleigh beams.

A natural extension of the work by Kambampati is to address Rayleigh beams, which contains the rotary inertia term. In this paper, we find (i) non-uniform rotating Rayleigh beams with continuous mass, mass moment of inertia and flexural stiffness distributions that are isospectral to a uniform non-rotating Rayleigh beam and (ii) non-uniform non-rotating Rayleigh beam isospectral to a uniform rotating Rayleigh beam of the same length under the same boundary condition as shown schematically in Fig. 1(a) and (b). We use a transformation to convert the non-dimensional Rayleigh beam equations from the (x, W) frame of reference to a hypothetical (z, U) frame of reference. If the material and geometric properties of the beam are specific chosen functions of the two introduced auxiliary variables, then the transformation will be achieved and, if the coefficients of the transformed equation match with the required one ((z, V) frame of reference), then the equivalence is established. We arrive at a pair of coupled ODEs which are solved for a particular case in which one of the auxiliary functions is constant. For case (i), we obtain the closed-form solutions of the mass per unit length, mass moment of inertia, centrifugal force and bending stiffness variations for a cantilever beam and hinged free beam with an elastic hinge spring, and for case (ii), we find non-uniform cantilever beams and beams with torsional spring at the free end isospectral to a given rotating uniform cantilever beam. The limiting value of rotational speeds for both the cases is also specified. The equivalence between the