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# Vibration of laminated composite quadrilateral plates reinforced with graphene nanoplatelets using the element-free IMLS-Ritz method

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ABSTRACT

This paper investigates free vibration of graphene nanoplatelet (GPL) reinforced laminated composite quadrilateral plates using the element-free IMLS-Ritz method. The effective material properties including Young's modulus, mass density and Poisson's ratio are determined by the modified Halpin–Tsai model and rule of mixture. The first-order shear deformation theory (FSDT) is employed for formulation of the energy functional. Based on the IMLS-Ritz approximation, the discrete vibration equation of the laminated composite quadrilateral plates is derived. The accuracy of the IMLS-Ritz results is examined by comparing with the published values. A comprehensive parametric study is carried out, with a particular focus on the effects of weight fraction, distribution pattern, geometry and size of GPL reinforcements, total number of layers and geometric parameters of quadrilateral plates on the natural frequencies of GPL reinforced laminated composite quadrilateral plate.

## 1. Introduction

Owing to the remarkable mechanical, electrical and thermal properties, carbon nanotube (CNT) reinforced nanocomposites have been widely studied in the past two decades [1]. Comparing with CNT reinforced nanocomposites, graphene or graphene nanoplatelets (GPLs) provide better interactive actions with the polymer matrix and can be potentially manufactured at low cost [2]. Recently, GPL reinforced composite (GPLRC) has been synthesized [3–5]. Researchers found that a small percentage of GPLs in composite can improve its mechanical, electrical and thermal properties substantially [4–7]. Comparing to carbon fiber reinforced composites which normally contain over 60% carbon fiber by volume, GPLRCs may only have a relatively low percentage of GPLs (0.5%–20% by weight) [8–10]. In view of these virtues, GPLRCs would have a wide range of engineering applications.

It is important to understand the mechanic characteristics of GPLRC structures. Chandra et al. [11] presented the natural frequencies and mode shapes of graphene/epoxy composite plates by using the finite element method (FEM) based on a multiscale approach. Rafiee et al. [12] experimentally studied the buckling capacity of graphene/epoxy composite beams. Parashar and Mertiny [13] investigated the buckling of graphene/epoxy composite plates by using FEM.

Recently, Yang and his co-authors studied various GPLRC beams and plates. They used the Halpin–Tsai micromechanics model and Timoshenko beam theory to obtain the mechanical model of GPLRC beams. The Ritz method was applied to study the nonlinear bending, buckling, postbuckling, and free vibration of GPLRC beams [14–18]. Bolotin's method was used to analyze the dynamic instability of GPLRC beams in thermal environment [19]. The differential quadrature (DQ) method was employed to investigate the buckling and postbuckling of GPLRC beams [20]. They also studied GPLRC plates by using the first-order shear deformation theory (FSDT). Navier's solution technique was used to analyze bending, buckling, free vibration and forced vibration of GPLRC plates [21,22]. The perturbation technique was also employed to determine the postbuckling of both perfect and imperfect GPLRC plates with simply supported on all edges [23]. The Mian and Spencer method was utilized to obtain the analytical solutions of GPLRC circular and annular plates [24] and thermoelastic bending behavior of GPLRC rectangular plate [25]. The FEM was employed to investigate the bending and vibration behaviors of GPLRC trapezoidal plates [26].

Based on Reddy's higher order shear deformation plate theory with von Kármán type nonlinearity, Shen and his co-authors studied nonlinear bending [27,28], nonlinear buckling and postbuckling [28–30], and nonlinear vibration [31–33] of functionally graded graphene-reinforced composite laminated plates resting on elastic foundations or in thermal environments by applying a two-step perturbation technique.

In addition, Barati and Zenkour [34] investigated the post-buckling behavior of geometrically imperfect porous GPLRC beams resting on nonlinear hardening foundation. Based on the sinusoidal shear deformation plate theory and von Kármán nonlinear relations, Gholami and

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Ansari [35] presented a large deflection geometrically nonlinear analysis of GPLRC rectangular plates. Sahmani and Aghdam [36] examined the nonlinear instability of GPLRC nanoshells under axial compressive load, applying a more comprehensive size dependent continuum elasticity theory. Kiani and Mirzaei studied the thermal buckling of GRC plates [37] and thermal postbuckling of GRC beams [38]. Kiani [39] also investigated the large amplitude free thermal vibration of GRC plates by using NURBS formulation.

A quadrilateral plate as an important structural component used in engineering has been widely studied. Al-Bermani and Liew [40] first reported the natural frequencies of thick arbitrary quadrilateral plates with different boundary conditions using the pb-2 Ritz method. The Ritz method is also widely used to study thermal buckling of FG-CNTRC plates [41], thermal postbuckling of FG-CNTRC beams [42], and free vibration of FG-CNTRC plates with cutout [43], FG-CNTRC skew plates [44] and FG-CNTRC spherical shell panels [45]. Karami and Malekzadeh [46] applied the DQ method to study free vibration of arbitrary straight-sided quadrilateral plates. Recently, the element free improved moving least-squares (IMLS) Ritz method [47,48] has been successfully developed to study free vibration of CNT reinforced composite plates resting on elastic foundations or with elastically restrained edges [49-55]. The element-free method may provide more accurate approximations for structures with complex geometries than FEM [56]. In contrast with other element-free methods, the IMLS-Ritz method proposed by Zhang et al. [47,48] could avoid forming an ill-conditioned or singular equation system, and need fewer nodes in the domain of influence. The computational advantages and special features of IMLS-Ritz method have been demonstrated through solving geometrically nonlinear large deformation [57–60], vibration [61,62] and dynamic responses [63] of quadrilateral plate.

Although some works have already been done on quadrilateral plates, there are only a few works which focused on free vibration behavior of GPLRC quadrilateral plates. In a major advance in 2017, Zhang [64] presented the first known implementation of the IMLS-Ritz method for dynamics analysis of laminated CNT-reinforced composite quadrilateral plates. This paper examined the remarkable features of the IMLS-Ritz method, which demonstrated to be efficiently used for solving vibration of laminated CNT-reinforced composite quadrilateral plates under in-plane forces. Zhang and Xiao [65] have further explored the IMLS-Ritz method for multilayered CNT-reinforced composite skew plates subjected to sudden transverse dynamic loads. Sets of useful benchmarking results for parametric studies of various CNT-reinforced composite plates were reported. These two papers [64,65] formed the fundamental of the present study by extending the IMLS-Ritz method for vibration analysis of GPLRC quadrilateral plates.

In this paper, the IMLS-Ritz method is employed to investigate free vibration of GPLRC quadrilateral plates. The modified Halpin–Tsai model is used to derive the effective Young's modulus, Poisson's ratio and mass density. The vibration equation of GPLRC quadrilateral plate is formulated based on the FSDT and the IMLS-Ritz procedure. The natural frequencies and mode shapes are obtained by solving the eigenvalue problem of vibration equation. Finally, the effects of graphene parameters and geometric parameters of plate on the dynamic characteristics of the GPLRC quadrilateral plate are investigated in detail.

## 2. Problem formulation

### 2.1. Material properties of GPLRCs

A multilayer quadrilateral plate under the current consideration is shown in Fig. 1. Each layer is of same thickness ($\Delta h = h/NL$, $NL$ is the total number of layers), but both the porosity coefficient and GPL volume fraction are graded from layer to layer. The effective Young's modulus, Poisson's ratio and mass density of the GPLRC quadrilateral plate are

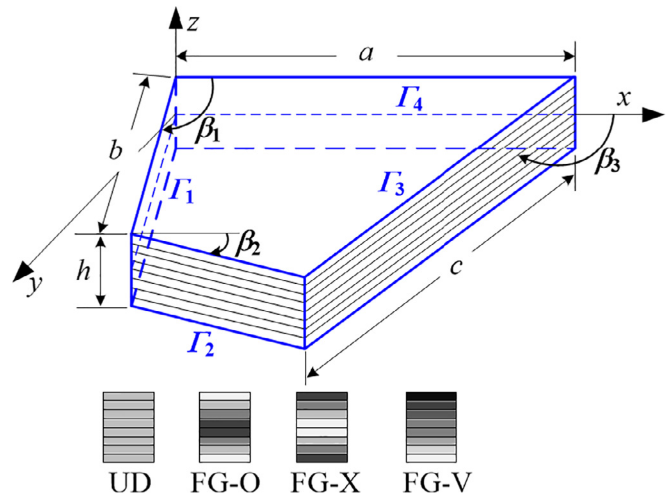


**Fig. 1.** Configurations of a GPLRC quadrilateral plate.

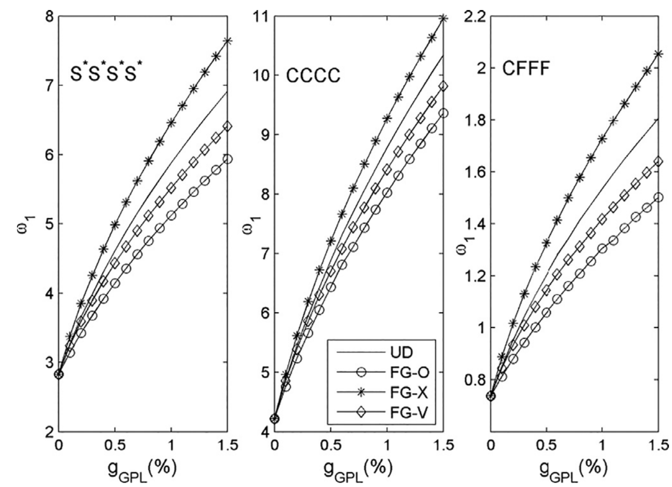


**Fig. 2.** The first order natural frequency of GPLRC quadrilateral plates with different $g_{GPL}$.

determined according to Halpin–Tsai model [66,67]

$$E_C = \frac{3}{8}\frac{1+\xi_L\eta_L V_{GPL}}{1-\eta_L V_{GPL}}E_M + \frac{5}{8}\frac{1+\xi_W\eta_W V_{GPL}}{1-\eta_W V_{GPL}}E_M \tag{1}$$

$$\nu_C = \nu_{GPL}V_{GPL} + \nu_M(1-V_{GPL}) \tag{2}$$

$$\rho_C = \rho_{GPL}V_{GPL} + \rho_M(1-V_{GPL}) \tag{3}$$

Subscripts "GPL", "M" and 'C' stand for GPLs, polymer matrix and GPL/polymer nanocomposite, respectively.

$$\eta_L = \frac{E_{GPL}-E_M}{E_{GPL}+\xi_L E_M} \tag{4}$$

$$\eta_W = \frac{E_{GPL}-E_M}{E_{GPL}+\xi_W E_M} \tag{5}$$

where $\xi_L$ and $\xi_W$ are the parameters characterizing both the geometry and size of GPL nanofillers, defined as

$$\xi_L = 2\frac{l_{GPL}}{h_{GPL}} \tag{6}$$

$$\xi_W = 2\frac{w_{GPL}}{h_{GPL}} \tag{7}$$