



# An effective length model for octet lattice

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## ABSTRACT

As technology advance in 3D printing, octet lattice materials can be made with a wide range of strut aspect ratios. It is essential to develop a satisfying theoretical model for large aspect ratio ranges. In this paper, an analytical model considering material overlapping effect in strut joint, bend and shear has been derived to predict the effect compressive stiffness and strength of the octet lattice materials with cylindrical struts. The models have been validated by FE simulations and experiments. These results demonstrate that the relative compressive stiffness and strength not only depend on the relative density, but also relate to the effect of strut joint, bend and shear. The material overlapping effect in strut joint and the coupling effect between the shearing force and the bending moment will raise the relative compressive stiffness and strength.

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## 1. Introduction

In recent years, lattice materials have been widely used in a broad range of applications such as load bearing structures and thermal insulation due to their outstanding specific stiffness and strength properties. Extensive efforts [1–33] have been devoted to investigating the mechanical properties of lattice materials.

Lattice materials can be categorized as either bending-dominated or stretching-dominated by topological criteria. The topological criteria had been investigated by Deshpande et al. [2]. Their study showed that the deformation mechanisms in lattice materials were determined by their nodal connectivity and the minimum nodal connectivity for 2D and 3D stretching-dominated lattice materials was 6 and 12 respectively.

The most common form of stretching-dominated lattice materials is octet lattice material. The effective mechanical properties of the octet lattice material were studied by Deshpande et al. [1] experimentally and theoretically. They found that for small  $r/l$ , the effect of bend on compressive stiffness could be negligible and the pin-jointed assumption sufficed. They stated that the effective mechanical properties were linearly scale with relative density. The relative compressive stiffness and strength were given by  $\bar{E} = \bar{\rho}/9$  and  $\bar{\sigma} = \bar{\rho}/3$  respectively, where  $\bar{\rho}$  was the relative density of the octet lattice material. A multi-surface plasticity model for the octet truss lattice materials was proposed by Mohr [34]. Based on early work, an increment equivalent continuum method [9] was developed by Fan et al. to study the nonlinear mechanical properties of lattice truss composite materials. Also, a computational homogenization approach is presented by Vigliotti et al. [25] to derive

a nonlinear constitutive model for lattice materials. In addition, Messner et al. [28] provided a dynamic continuum model to simulate the nonlinear behavior of the octet truss material.

As technology advance in 3D printing, octet lattice materials can be made with a wide range of strut aspect ratios. A parametric finite element study was carried out by Tancogne-Dejean et al. [31] to determine the effect of relative density and strut aspect ratios on the relative compressive stiffness. They found that the relative compressive stiffness for relative density higher than 0.1 scaled with a power exponent higher than 1. Obviously, the ideal linear model gives quite low predictions for large ratio ranges. Hence, there is a need to develop a satisfying theoretical model for large aspect ratio ranges.

In the present work, an analytical model considering the material overlapping effect in strut joint and the coupling effect between the shearing force and the bending moment has been developed to calculate the effective compressive stiffness and strength, which has been validated by finite element simulations and experiments. The effects of strut joint, bend and shear on the relative compressive stiffness and strength have been discussed finally.

## 2. Relative density of octet lattice material and equivalent length of strut

A unit cell of octet lattice material is shown in Fig. 1b, which can be stacked in all principal directions to construct the entire structure's geometry shown in Fig. 1a. The unit cell consists of 36 struts with same length  $l$ . For a perfectly cylindrical strut with a constant radius  $r$ , a first-

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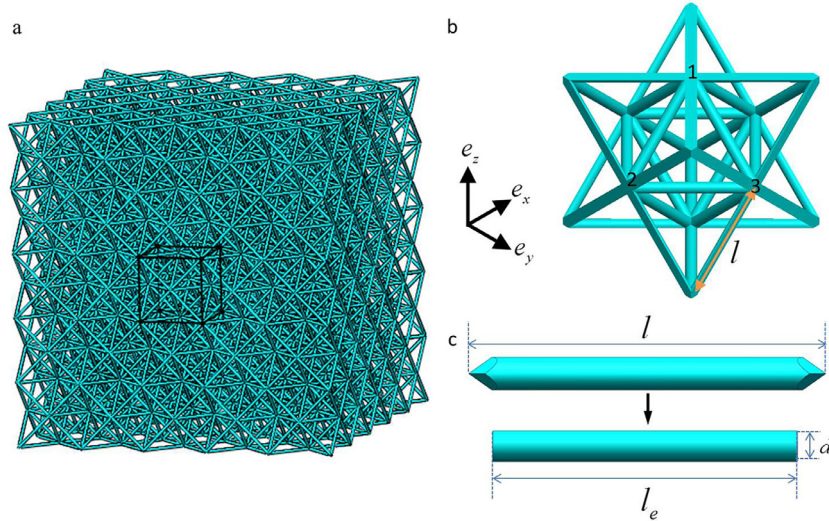


Fig. 1. (a) octet truss lattice material, (b) representative unit cell and (c) geometry of individual strut.

order approximation of the relative density [1] can be given by

$$\bar{\rho} = 6\sqrt{2}\pi\left(\frac{r}{l}\right)^2 \quad (1)$$

However, the above analytical relationship is only valid for low relative density due to strut joint effect. To mathematically analyze octet unit cell, the actual geometry of individual strut need to be taken into account. As is shown in Fig. 1c, both ends of micro strut have very complex and non-smooth surfaces. So it is difficult to exactly determine the true radius of the strut. Therefore, in this study the strut radius is assumed to be constant and an effective strut length will be calculated using the definition of volume of micro strut. The effective strut length will be used in the following theoretical studies.

Consider the overlapping parts of struts in the unit cell, the actual volume of each strut can be obtained by integration as

$$V_l = \pi r^2 l_e \quad (2)$$

where the effective strut length  $l_e$  can be written as:

$$l_e = \left(1 - \left(\sqrt{2} + \frac{2}{\pi}\right)\frac{r}{l}\right)l \quad (3)$$

It can be shown that the equivalent length is determined by the diameter to length ratio of struts. When the aspect ratio is small enough, the equivalent length  $l_e$  can be approximated by strut length  $l$ . If the aspect ratio tends to be large, the equivalent length will be far less than the strut length and the strut joint effect must be considered in the relative density calculation.

As some struts in a unit cell are shared by adjacent cells, the total volume of strut in the unit cell is  $V = 24\pi r^2 l_e$ . Here, the volume of unit cell is

$$V_c = 2\sqrt{2}l^3 \quad (4)$$

Then, the relative density of octet lattice material can be written as

$$\bar{\rho} = \frac{V_c}{V} = 6\sqrt{2}\pi\left(\frac{r}{l}\right)^2 \frac{l_e}{l} = 6\sqrt{2}\pi\left(\frac{r}{l}\right)^2 \left(1 - \left(\sqrt{2} + \frac{2}{\pi}\right)\frac{r}{l}\right) \quad (5)$$

As shown in Fig. 2, the evolutions of relative density as a function of strut aspect ratio  $r/l$  are given for both Eqs. (1) and (5). To compare the effectiveness of two equations, the CAD predictions for the relative density are also given in same figure. From the figure, Eq. (1) gives increasing errors compared to the CAD predictions as the aspect ratio increases. Inevitably, the prediction errors will affect the accuracy of theoretical prediction for relative compressive stiffness and strength, which are of critical importance for cellular materials. Accounting for the strut joint effect, Eq. (5) can give better results especially for large aspect ratio.

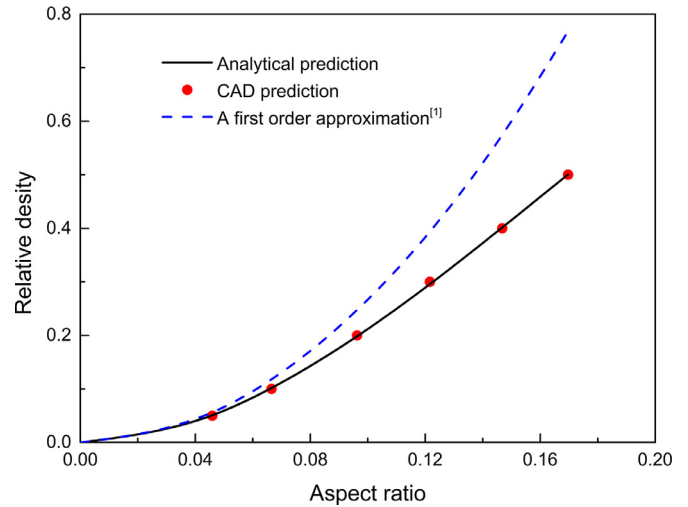


Fig. 2. Evolutions of relative density as a function of strut aspect ratio.

### 3. Analytical model

#### 3.1. Determination of the compressive stiffness

In this section, a theoretical approach for predicting the compressive stiffness and strength for octet lattice is introduced. As can be seen from Fig. 1c, the geometric structure of micro strut is very complex and the traditional theoretical analysis based on the bending beam theory is not applicable to this case. To be able to use the theory of bending beam, in this study the strut radius is assumed to be constant. Based on the conservation of mass and conservation of energy, the effective strut length  $l_e$  will be utilized for energy computations instead of the actual strut length  $l$ .

As a result of symmetry, the struts shown in Fig. 3a can be divided into three types, which are labeled as 12, 13 and 23, according to their spatial direction. When the octet lattice structure is under uniaxial compression, both ends of micro struts can be able to move in three main directions. The struts can be considered as doubly clamped beams subjected to axial stretching force, bending moment and shear load.

Considering the doubly clamped beam 12 shown in Fig. 3, the relative displacement between 1 and 2 can be determined by unknown global displacement  $u$  and  $w$  as  $(u, 0, w)$ . The axial displacement  $\delta_1$  of

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