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Functionally graded microbeams: Simultaneous presence of imperfection and viscoelasticity

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ABSTRACT

As the first endeavour, the coupled nonlinear mechanical behaviour of extensible functionally graded microbeams, when both viscoelasticity and imperfection are present, is investigated. The imperfect viscoelastic microbeam is subject to a transverse harmonic excitation load of a constant amplitude. The Kelvin–Voigt viscoelastic model and Mori–Tanaka homogenisation method are used together in order to describe the internal energy loss and the variation of the material properties of the microsystem along the transverse direction, respectively. The geometric imperfection is modelled by imposing an initial curvature in the transverse deformation of the viscoelastic microscale beam. Using the Euler–Bernoulli strain–displacement relations, the geometric nonlinearity is taken into account. The non-classical nonlinear equations of motion are derived on the basis of Hamilton's principle and the modified couple stress theory. The resulting equations are found to be coupled between transverse and longitudinal oscillations. Galerkin's technique and the method of pseudo-arclength continuation as well as direct time-integration approach are finally employed to solve the governing differential equations for oscillation frequencies and nonlinear dynamic response. It is found that the nonlinear forced oscillations of extensible functionally graded microbeams are greatly affected by the internal energy loss together with geometric imperfection. It is shown that the simultaneous presence of viscoelasticity and geometric imperfections governs both the amplitude and softness/hardness of the dynamical behaviour.

1. Introduction

In recent years, small-scale structures such as micro/nanoscale beams and plates have attracted an extensive interest due to their unique mechanical and physical properties [1,2]. Micro/nanoscale structures have a wide range of applications in the new fields of engineering such as micro/nanomechanics and biomechanics [3,4]. In particular, micro/nanoscale beams including carbon nanotubes (CNTs), ZnO nanowires, boron nitride nanotubes (BNNT) and silicon microbeams have been used as a building block of many microelectromechanical systems (MEMS) and nanoelectromechanical systems (NEMS). For instance, in an interesting work, a label-free single-cell analysis was carried out using ultrasound microscale beams [5]. On the other hand, to improve the mechanical performance of micro/nanoscale beams, the new generations of this class of microsystems have been manufactured of inhomogeneous composites such as functionally graded (FG) materials [6]. In fact, microstructures made of FG materials [7,8] are more resistant to thermal and mechanical stresses than the conventional homogeneous structures. These alluring features make FG microstructures favourable in the generation of novel MEMS devices.

Since the mechanical characteristics of micro/nanoscale materials are size-dependent, the use of the traditional classical continuum mechanics in the static and dynamic analysis of these materials might be uncertain. Over the two past decades, some modified size-dependent elasticity theories [9,10] have been introduced to overcome the shortcomings of the traditional elasticity theories. Eringen's nonlocal elasticity theory [11–13], strain gradient theory [14,15], the theory of couple stress [16–18] and nonlocal strain gradient elasticity theory [19,20] are the most popular ones due to their simplicity as well as their capability in the modelling of complex structures at the micro/nanoscale level. Among these higher-order size-dependent theories, the modified version of the couple stress theory and strain gradient theory have been used by many researchers [21–36] in order to study the mechanics of microstructures. However, the mechanical behaviour of nanostructures is commonly analysed by means of the nonlocal elasticity theory. Wang et al. [37] used the nonlocal version of the Timoshenko beam theory in order to examine the bending of micro and nanobeams. Danesh et al. [38] presented a nonlocal continuum model for the axial oscillation of tapered nanorods with various boundary conditions. They employed the differential quadrature method as a numerical tool to determine the fundamental frequencies of the non-uniform nanorod. In an another work, Akgöz and Civalek [39] investigated the longitudinal oscillation of microbars based on the theory of strain gradient elasticity. In addition, a size-dependent mathematical model was proposed by Li et al. [40] for

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the deformation of a bi-layered micro-plate under an applied load via the strain gradient theory. Roque et al. [41] employed the Timoshenko beam theory and the modified couple stress theory so as to develop a continuum model for the bending of laminated composite nanobeams subject to transverse loads. They solved the governing differential equations of the system numerically via a meshless technique. Ghayesh et al. [42] examined the nonlinear dynamic response of microbeams in the context of the theory of couple stress elasticity.

Besides the development of modified continuum models for CNTs, and piezoelectric nanobeams, size-dependent elasticity theories have been also employed to investigate the statics and dynamics of FG microstructures. Şimşek and Yurtcu [43] presented a closed-form solution for the bending and buckling of FG nanoscale beams within the framework of a nonlocal beam theory. Moreover, the strain gradient theory and the Euler–Bernoulli beam theory were used to examine the buckling behaviour of size-dependent functionally graded microbeams [44]. An exact solution was also obtained for the large-amplitude forced oscillation of nanoscale beams made of FG materials in the presence of thermal effects via use of a surface elasticity theory [45]. Furthermore, Thai et al. [46] used a modified strain-gradient theory to explore the nonlinear response of FG microplates subjected to static and dynamic loadings using an isogeometric approach. In another research study, a consistent couple-stress model was developed for the oscillation of nanobeams made of arbitrary bi-directional FG materials [47].

Viscosity is one of effective ways to model internal energy dissipation (friction) in continuous structures such as beams and plates; this effect can be dominant in the mechanical behaviour of microsystems [48,49]. The Kelvin–Voigt viscosity model is used in this study to include the internal friction effect. In practical applications, there may be a *geometric imperfection* in micro/nanoscale structures, which can cause a considerable change in their mechanical characteristics. Such initial imperfections can be caused by initial stress or the lack of sufficient precision in the manufacturing process.

According to the best of my knowledge, the simultaneous effects of geometric imperfections and viscosity on the nonlinear forced oscillation of extensible FG microbeams have not been studied yet. This motivates me to present a scale-dependent continuum model for this problem. The length-scale effect is taken into account via the modified couple stress theory. In order to model the internal energy dissipation of the system, the Kelvin–Voigt viscoelasticity is incorporated into the relations of stress tensor and the symmetric couple stress tensor. Using von Karman's theory, the geometric nonlinearity is considered in the formulation. Both the longitudinal and transverse displacements as well as the inertia of the viscoelastic FG imperfect microbeam are considered in the coupled dynamic analysis. The geometric imperfection of the microscale system is caused by an initial deflection in the thickness direction. The nonlinear curves of the dynamics of extensible FG viscoelastic imperfect microbeams are obtained employing the Galerkin procedure together with the pseudo-arclength continuation approach in conjunction with the method of direct time-integration. The numerical results are presented for various viscosity and imperfection values in order to highlight their effects.

2. Model development for the viscoelastic imperfect microsystem and discretisation

In this section, a modified continuum model is developed for the coupled nonlinear dynamics of functionally graded (FG) viscoelastic microbeams with geometric imperfections. Fig. 1 shows an extensible FG microscale beam with viscoelastic properties under a harmonic transverse loading $F(x)\cos(\omega t)$ where ω, $F(x)$ and t stand for the excitation frequency, forcing amplitude and time, respectively. Let me denote the displacements in the x and z directions by u and w, respectively. As can be seen from the figure, an initial deflection w_0 is taken into account in the transverse direction, as an initial imperfection [50]. In the present work, the microscale beam is composed of a ceramic-metal mixture so that effective physical and mechanical properties vary along the thickness direction. Therefore, for the approximation of the effective material properties of the microsystem, the Mori–Tanaka law may be used [51]. Based on this approach, we have

$$\begin{aligned}\frac{\mu-\mu_m}{\mu_m-\mu_c} &= \frac{-\upsilon_c}{1-\upsilon_m(\mu_m-\mu_c)/[\mu_m+\mu_m(8\mu_m+9K_m)/(6(2\mu_m+K_m))]},\\ \frac{K-K_m}{K_m-K_c} &= \frac{-\upsilon_c}{1-\upsilon_m(K_m-K_c)/(4\mu_m/3+K_m)},\end{aligned} \tag{1}$$

where K, ν and μ are, respectively, the effective values of bulk modulus, volume fraction and shear elasticity modulus. In addition, in the above relations, m and c denote the metal and ceramic phases, respectively. The simple relationship between the two volume fractions of FG microbeams is described as

$$\upsilon_c+\upsilon_m=1. \tag{2}$$

The distribution of the ceramic across the thickness is usually assumed as follows [51]

$$\upsilon_c(z)=(0.5+z/h)^n. \tag{3}$$

Here h and n are the thickness of the microbeam and the gradient index, respectively. It should be noted that the effective elasticity modulus (E) and Poisson's ratio (ν) also vary in the z direction and can be calculated as

$$E(z)=\frac{9K\mu}{3K+\mu}, \tag{4}$$

$$\nu(z)=\frac{3K-2\mu}{6K+2\mu}. \tag{5}$$

In a similar way, for other material properties, one obtains

$$\rho(z)=\rho_m\upsilon_m+\rho_c\upsilon_c, \tag{6}$$

$$l(z)=l_m\upsilon_m+l_c\upsilon_c, \tag{7}$$

$$\eta(z)=\eta_m\upsilon_m+\eta_c\upsilon_c, \tag{8}$$

where ρ, l and η represent the effective mass density, the length-scale parameter and the viscosity coefficient, respectively. In the present work, it is assumed that the cross-sectional area of the microbeam is constant, and it remains perpendicular to the middle plane. Moreover, only geometric nonlinearity induced by the stretching of the mid-plane is considered. The nonlinear strain–displacement relation of an Euler–Bernoulli microbeam with a geometric imperfection can be written as

$$\varepsilon_{xx}=\frac{\partial w}{\partial x}\frac{\mathrm{d}w_0}{\mathrm{d}x}-z\frac{\partial^2 w}{\partial x^2}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial u}{\partial x}, \tag{9}$$

where ε_{xx} represents the strain component along the x axis and perpendicular to the cross-sectional area of the microbeam. It is worth mentioning that other strain components are equal to zero (i.e. $\varepsilon_{xz}=\varepsilon_{zx}=\varepsilon_{yy}=\varepsilon_{zz}=\varepsilon_{xy}=\varepsilon_{yz}=0$). According to the modified theory of couple stress in conjunction with the Euler–Bernoulli theory of beams we have [52]

$$\begin{aligned}\theta_y &= -\frac{\partial w}{\partial x},\\ \theta_x &= \theta_z=0,\end{aligned} \tag{10}$$

and

$$\begin{aligned}\chi_{xy} &= -\frac{1}{2}\frac{\partial^2 w}{\partial x^2}\\ \chi_{yx} &= -\frac{1}{2}\frac{\partial^2 w}{\partial x^2}.\end{aligned} \tag{11}$$