International Journal of Mechanical Sciences 140 (2018) 394–406

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

An improved method of determining Gotoh's nine material constants for a sheet metal with only seven or less experimental inputs

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ARTICLE INFO

Keywords:
Anisotropic plasticity
Non-quadratic yield criterion
Orthotropic flow potential
Sheet metal forming
Intrinsic variables
Convex polynomials

ABSTRACT

A method is described for estimating the material constants in Gotoh's fourth-order yield stress function for modeling anisotropic yielding and plastic flow of a sheet metal with only up to seven available experimental inputs. The yield function is assumed to have a reduced degree of planar anisotropy so its nine material constants can be computed directly via simple algebraic relations using seven or less experimental inputs available for the sheet metal. The key idea behind the proposed method is to first transform a polynomial yield function such as Gotoh's in terms of two principal stresses and one loading directional angle and to set coefficients of some high order Fourier cosine terms in the transformed yield function to be zero as needed. Not only the method is much simpler to use in comparison with many other existing methods appeared in the literature, the results obtained for many representative sheet metals also give consistent and even superior description of directional dependence of uniaxial tensile yield stress and plastic strain ratio without any undesired spurious and large oscillations. As Gotoh's yield function is one of the simplest complete non-quadric polynomials with the enhanced modeling capability, the simple analytical calculations of all of its material constants should facilitate its wider application as the new default choice of orthotropic plasticity modeling in industrial sheet metal forming analyses.

1. Introduction

The classical associated plasticity theory for describing the yielding and plastic flow of orthotropic sheet metals was first proposed by Hill in 1948 [1] and the plane stress version of his quadratic polynomial yield function has the following form

$$\Phi_h(\boldsymbol{\sigma}) = \bar{\sigma}^2 = F\sigma_y^2 + G\sigma_x^2 + H(\sigma_x - \sigma_y)^2 + 2N\tau_{xy}^2, \tag{1}$$

where $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \tau_{xy})$ is the Cartesian stress with its components in sheet metal orthotropic axes (with the *x*-axis being corresponding to the rolling direction of a sheet metal), *F, G, H,* and *N* are its four material constants, and $\bar{\sigma}$ may be regarded as an equivalent yield stress of the sheet metal undergoing plastic deformation.

As Hill's quadratic yield function has been found over the years to be inadequate, non-quadratic yield functions are increasingly used instead for associated plasticity modeling of many sheet metals in industrial forming applications [2–4]. One of the earliest and also simplest non-quadratic yield functions is the fourth-order homogeneous polynomial studied by Gotoh in 1977 [5], i.e.,

$$\Phi_g(\boldsymbol{\sigma}) = A_1\sigma_x^4 + A_2\sigma_x^3\sigma_y + A_3\sigma_x^2\sigma_y^2 + A_4\sigma_x\sigma_y^3 + A_5\sigma_y^4 + A_6\sigma_x^2\tau_{xy}^2 + A_7\sigma_x\sigma_y\tau_{xy}^2 + A_8\sigma_y^2\tau_{xy}^2 + A_9\tau_{xy}^4, \tag{2}$$

where A_1, A_2, ... and A_9 are its nine material constants. Gotoh [5,6] detailed in particular the parameter identification of those nine material constants for two sheet metals using one measurement σ_b from an equal biaxial tension test and eight measurements (σ_0, σ_{45}, σ_{90}, σ_θ, R_0, R_{45}, R_{90}, R_θ, where $\theta = 22.5^\circ$ or 67.5°) from at least four uniaxial tension tests.

Gotoh's 1977 yield function captures better the directional variation of both yield stresses and plastic strain ratios under uniaxial tension and may model the so-called *anomalous behavior* under equal biaxial tension. Some recent investigations indicated that Gotoh's yield function is sufficiently robust and sometimes even superior than other non-quadratic or non-associated anisotropic plasticity models of lightweight aluminum alloys and advanced high strength steels [7–10]. Nevertheless, with few exceptions [11–13], Gotoh's yield function has however not been more widely considered for sheet metal forming analysis applications outside Japan since its initial developments. Among the plausible reasons given in the past for such a situation include 1) Gotoh's yield function in its form presented in Eq. (2) is quite complicated and its material constants have no physical meaning [14,15]; 2) the polynomial function is not guaranteed in general to be strictly positive and convex [13,16]; and 3) it has too many material constants [15] as usually only up six to eight mechanical property measurements (σ_0, σ_{45}, σ_{90}, σ_b, R_0, R_{45}, R_{90}, R_b)

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https://doi.org/10.1016/j.ijmecsci.2018.03.018
Received 10 August 2017; Received in revised form 6 March 2018; Accepted 14 March 2018
Available online 16 March 2018
0020-7403/

NOTATION

x,y,z	The orthotropic material symmetry axes corresponding to the rolling (RD), transverse (TD), and normal (ND) directions of a thin sheet metal.
σ_x, σ_y, τ_{xy}	Three in-plane Cartesian (two normal and one shear) components of an applied Cauchy stress in the orthotropic coordinate system of the sheet metal.
Φ_h, F,G,H,N	Hill's 1948 quadratic anisotropic yield stress function in plane stress and its four material constants.
Φ_g, A_1, ... ,A_9	Gotoh's 1977 fourth-order anisotropic yield stress function in Cartesian stress components (σ_x, σ_y, τ_{xy}) and its nine material constants.
σ_θ, R_θ, σ_b, R_b	Yield stress and plastic strain ratio under uniaxial tension at the loading orientation angle θ and under equal biaxial tension.
σ_1, σ_2, θ	The so-called intrinsic variables according to R. Hill, namely, the two in-plane principal stresses (σ_1, σ_2) and the loading orientation angle θ between σ_1 and the RD of the sheet metal.
ϕ_g, $F(\theta)$, $G(\theta)$, $H(\theta)$	Gotoh's yield function recast in intrinsic variables in terms of five homogeneous stress terms (σ_1^4, $\sigma_1^3\sigma_2$, $\sigma_1^2\sigma_2^2$, $\sigma_1\sigma_2^3$, σ_2^4) and its three in-plane Fourier cosine series functions in terms of θ. Those three functions have a total of nine independent coefficients (F_0, F_1, F_2, F_3, F_4, G_0, G_1, G_2, H_0).

are most often reported for a sheet metal in developing its non-quadratic yield functions [15,17–19].

The first two objections or difficulties about using Gotoh's yield function have recently been effectively dealt with. Firstly, a yield function expressed in terms of a *complete* polynomial form such as Eq. (2) is now actually called "user-friendly" and is increasingly preferred in a practical finite element coding implementation [16]. That is, many existing homogeneous yield functions of a high integer order can always be rewritten into such a *unique* and *consistent* polynomial form [16,20,21]. The material constants (the polynomial coefficients) can simply be obtained from a set of analytically derived *linearly* independent equations in terms of various yield stress ratios of the same integer order and plastic strain ratios [5,13,16,21,22].

Secondly, although a yield function in terms of a homogeneous polynomial such as either Hill's 1948 yield function Φ_g or Gotoh's 1977 yield function Φ_g is not guaranteed in general to be unconditionally positive and convex, one can always check if a *calibrated* polynomial yield function with known coefficients is strictly positive and convex or not. Hill's quadratic yield function has its unique advantage on this regard as its positivity and convexity can be easily verified using the necessary and sufficient conditions in terms of simple algebraic inequalities on its material constants as given by Hill in [23]. No such simple algebraic inequalities on the nine material constants of Gotoh's yield function have been reported so far as necessary and sufficient conditions that ensure the yield function is positive and convex. Nevertheless, there is no actual uncertainty regarding the positivity and convexity of a specific Gotoh's yield function if it has been fully calibrated for a given sheet metal: it has recently been shown that its strict positivity and convexity can always be straightforward checked and established *numerically* in several ways [10,21]. The calibrated Gotoh's yield function with certified positivity and convexity can thus be safely used in a rate-independent and associated plasticity framework for plastic strain increment calculations of the sheet metal under biaxial loading.

This study focuses on the third and remaining issue that may hinder a wider application of Gotoh's yield function when the results of only up to three uniaxial tensile tests and one equal biaxial tension test are used to characterize the anisotropic yielding and plastic flow of a sheet metal. There are at least four different approaches that have been reported in the literature for parameter identification of Gotoh's yield function when only seven experimental measurements (σ_0, σ_{45}, σ_{90}, σ_b, R_0, R_{45}, R_{90}) are made available for a sheet metal. The first approach simply assumes that two of the nine material constants A_6 and A_8 in Gotoh's yield function are the same and both are equal to $-A_7$ [24]. Its shortcoming of predicting spurious and unrealistically large oscillations in uniaxial plastic strain ratios in some sheet metals is well-documented [25]. The second approach is to fill in the two missing experimental inputs from additional tensile tests by averaging [12] or interpolating the available uniaxial tensile yield stresses or plastic strain ratios [10]. The third approach belongs to the classical optimization method using weighted least-squares of both yield stress differences and plastic strain ratio differences for parameter identification when the number of experimental inputs is equal to or exceeds the number of independent material constants in a yield function [13,16,18,26]: the specific example related to Gotoh's yield function was appeared in [13] which uses a weighted average method instead of a simple average method [24] to fill in four missing experimental inputs from two additional uniaxial tensile tests and then minimizes a least square error to obtain values of A_6 and A_8 from a set of eleven inputs (the four filled-in inputs plus the seven actual experimental inputs). The fourth approach is to use the well-known non-quadratic yield function YLD2000-2D that needs only up to eight experimental inputs for its parameter identification [27]: when the stress exponent $M = 4$ is used by Kuwabara et al. [8], their YLD2000-2D is basically another reduced form of Gotoh's yield function.

We present here yet another but more rational and much simpler approach towards the parameter identification of Gotoh's yield function for a sheet metal with only up to seven experimental inputs. The idea is to explicitly assume the sheet metal with a reduced planar anisotropy or equivalently to drop out higher order sinusoid terms from its Gotoh's yield function. These conditions of reduced anisotropy constitute naturally as additional constraints on Gotoh's nine material constants so the total number of independent material constants matches the number of available experimental inputs for the sheet metal. This parameter identification approach of using only limited and insufficient experimental inputs by setting some higher order Fourier coefficients to be zero is generally applicable to other higher-order *polynomial* yield functions as well although the analytical expressions for each polynomial coefficient in those yield functions may not be as simple and short as the case of Gotoh's yield function studied here. That is, the solution of linear equations analytically derived via the proposed parameter identification approach may be obtained numerically for all coefficients in practice in those higher-order polynomial cases.

In Section 2, the new parameter identification method is detailed by re-casting first the original Gotoh's yield function in terms of principal stresses and Fourier cosine series functions of the in-plane loading orientation angle [21,22]. Being consistent with the limited availability of experimental measurement inputs for a sheet metal with seven, five and three experimental inputs, the method sets its corresponding two, four or six higher-order Fourier coefficients in the transformed yield function to be zero accordingly. All of nine polynomial coefficients (material constants) in the original Gotoh's yield function can then be obtained via simple algebraic expressions using seven or less experimental inputs. Results of positive and convex Gotoh's fourth-order yield functions are given in Section 3 by applying the proposed parameter identification method to a total of nine representative sheet metals with only seven experimental measurement inputs. Gotoh's yield functions calibrated for first six sheet metals using both the newly proposed and several existing parameter identification methods appeared in the literature are directly