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# A new method based on energy principle to predict uniaxial stress–strain relations of ductile materials by small punch testing

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ABSTRACT

Uniaxial stress–strain relations of ductile materials are the most fundamental requirement for the design and safety assessment of structures. In this paper, a novel small punch testing (SPT)-related stress–strain relation (SPT-SR) model is proposed to predict the stress–strain relations of materials on the basis of equivalent energy principle. To examine the validity of the model, numerical simulations by finite element analysis (FEA) with a series of hypothetical materials with different Hollomon's material parameters were carried out. The results demonstrate that the uniaxial stress–strain relations predicted by SPT-SR model from simulated load-displacement curves are in good agreement with the properties of hypothetical materials used in FEA. Furthermore, SPT experiments and tensile tests for four steels (P92, DP600, Q345B, A508-III) at room temperature and 300 °C were conducted. The stress–strain relations predicted by SPT-SR model agree well with the tensile results generated from finite-element-analysis aided testing (FAT) method. Compared with the tensile strengths obtained by uniaxial tensile tests, they can also be well determined based on the strength coefficient and strain hardening exponent predicted by SPT-SR model.



## 1. Introduction

As one of the most important mechanical properties of materials, the uniaxial stress–strain relation is usually found by traditional material tensile test, which are difficult to apply in small-size components or in-service structures. Small punch testing (SPT) [1,2] was first suggested to obtain mechanical properties of in-service nuclear materials in 1980s. In the past thirty years, SPT has been developed for its extended capability for testing material properties of small-size specimens [3–8]. According to the SPT experimental results of five materials with different shapes and thicknesses, Mao [3] proposed two empirical relations between yield punching load of SPT and the tensile yield strengths, as well as between the maximum load of SPT and the tensile strengths. Many other researchers [9–14] have established similar empirical formulas based on their own experimental results in the same manner. However, all those empirical expressions in SPT method were quite different because they were determined by numerical fit of experimental data, and the definition of yield punching load was not yet unified [15]. In addition, other researchers have obtained the mechanical properties of materials by SPT based on either neural networks [16] or inverse analysis [17–19], but massive calculations by finite element analysis (FEA) were required to build the database or to perform an inverse iteration procedure. In 2006, a European Code of SPT method [20] was published, in which the experimental method was specified in detail. With reference to the European Code, Chinese standardization organization issued the test standard of SPT method [21] in 2012. Nevertheless, it's still a difficult process to obtain the stress–strain relations of materials using the load-displacement curves of SPT theoretically.

In this paper, based on the Chen–Cai energy principle [22], a SPT-related stress-strain relation (SPT-SR) model for predicting stress–strain relations of materials with Hollomon law by SPT is proposed and its validity checks are given.

## 2. Research conditions

### 2.1. Materials and experiment

Four steels (P92, DP600, A508-III and Q345B) were used for SPT experiments; their mechanical properties at room temperature and 300 °C are listed in Table 1. The split type of SPT experimental device (see Fig. 1) was adopted in this paper. The diameter $D$ and thickness $B$ of the SPT specimen were 10 mm and 0.5 mm, respectively. The steel ball radius $r$ was 1.25 mm and the lower die hole diameter $d$ was 4 mm, with 0.5 mm chamfer edge. The typical load-displacement ($P$-$h$) curve of SPT is shown in Fig. 2 whose first two stages bear bending, the

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**Table 1**
Mechanical properties of the four materials.

| Temperature | Materials | Elastic modulus $E$/MPa | Yield strength $\sigma_{p0.2}$/MPa | Tensile strength $\sigma_b$/MPa | Strength coefficient $K$/MPa | Hardening exponent $N$ |
|---|---|---|---|---|---|---|
| Room temperature | P92 | 212,000 | 548.4 | 686.1 | 956.1 | 0.102 |
| | DP600 | 183,000 | 348.1 | 593.8 | 1017 | 0.196 |
| | A508-III | 206,000 | 375.9 | 628.7 | 921.2 | 0.124 |
| | Q345B | 217,000 | 316.5 | 519.8 | 851.7 | 0.181 |
| 300 °C | A508-III | 182,000 | 468.4 | 628.4 | 903.4 | 0.125 |
| | Q345B | 197,000 | 483.8 | 582.3 | 775.1 | 0.0869 |

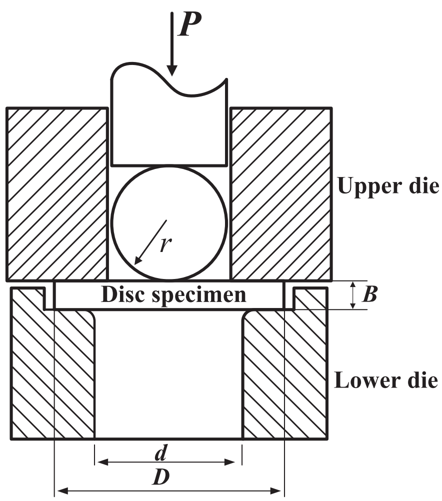


**Fig. 1.** Schematic illustration of SPT apparatus.

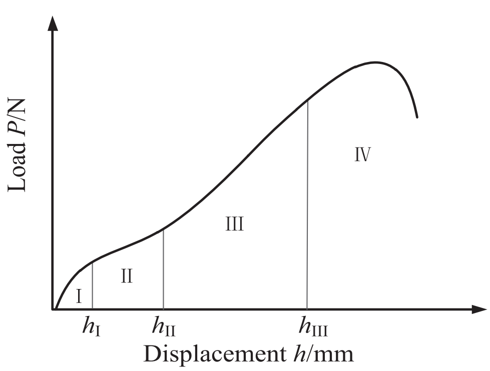


**Fig. 2.** Typical load-displacement curve of small punch test.

third stage mainly bears tensioning, and the fourth stage is tensioning up to failure. Following the European standards [20] (See Section 6.3, Part B), the starting displacement location of third stage for the load-displacement curve of SPT is recognized approximately as the displacement that reaches the magnitude of specimen thickness, i.e. the starting displacement $h_{II}$ can be taken as 0.5 mm for our disc. To eliminate the effect of roughness on the experimental results, the surface of each disc specimen was polished.

The SPT experiments were performed by using MTS809 25 k N electro-hydraulic servo testing machine with TestStarII controller system whose systematic error was less than 1%, and a constant displacement rate of 0.5 mm/min was employed. MTS 632.03F-30 COD extensometer and MTS 632.68F-08 strain extensometer were used to measure the displacement of SPT specimen at room temperature and 300 °C respectively.

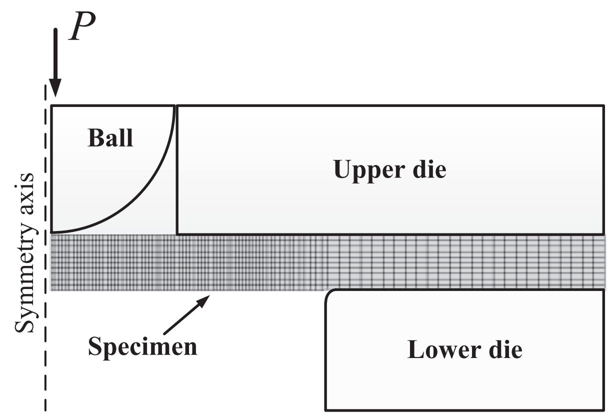


**Fig. 3.** Axisymmetric finite element model of SPT.

### 2.2. Numerical simulations

Fig. 3 shows the 2D axisymmetric finite element model of SPT specimen. The material was assumed to satisfy the uniform continuous, isotropic hardening and von–Mises equivalence. The upper die, lower die, and steel ball were considered as rigid body, which was simulated by target169 rigid element. The specimen was simulated by solid182 element and the surface of the specimen was treated as the slave surface of contact using contact172 element. The friction coefficient $\mu$ was set to 0.1. 1368 4-node axisymmetric elements and 1426 nodes were included in the model.

### 2.3. FAT method

The traditional uniaxial tensile method can only obtain the true stress–strain curves of materials before necking. In recent years, a new method of finite-element-analysis aided testing (FAT) has been proposed by Yao et al. [23,24], where the full-range uniaxial stress–strain relation of material up to failure can be obtained by tensile test of a notched round bar specimen. In this study, the full-range stress–strain relations of P92, DP600, A508-III and Q345B at room temperature, as well as A508-III and Q345B at 300 °C, were obtained by FAT method, as shown in Figs. 4 and 5. These relations compared with the results predicted by SPT-SR model will be discussed in the later section.

## 3. The theoretical model

It is well known that plastic deformation of components can be predicted based on the uniaxial stress–strain relation of representative volume element (RVE) of materials under proportional loading condition. The Hollomon law is commonly used to describe the elasto-plastic constitutive relationship of most ductile materials, which is expressed as follows

$$\sigma = \begin{cases} E\varepsilon & \sigma \le \sigma_y \\ K\varepsilon^n & \sigma \ge \sigma_y \end{cases} \tag{1}$$