International Journal of Mechanical Sciences 137 (2018) 238–251

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

On vibration transmission between interactive oscillators with nonlinear coupling interface

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ARTICLE INFO

Keywords:
Power flow analysis
Vibration transmission
Nonlinear stiffness
Chaos
Nonlinear interface
Vibration energy transfer

ABSTRACT

This paper investigates the dynamic characteristics and vibration transmission behaviour of interactive oscillators with nonlinearities at their coupling interface. Three different types of stiffness nonlinearities, i.e., hardening stiffness, softening stiffness and double-well potential type stiffness and cubic damping nonlinearity are considered. Both analytical approximations based on the method of averaging and also numerical integrations are employed to obtain the steady-state response and to determine the vibration transmission level. The time-averaged vibration power variables and kinetic energies of the system and the force transmissibility are formulated and obtained analytically and numerically. Time-averaged transmitted power is used as an index to quantify vibration transmission associated with both periodic responses and non-periodic responses such as chaos. It is found that hardening stiffness nonlinearity at the interface can lead to higher vibration power transmission at high excitation frequencies. In comparison, softening stiffness nonlinearity at the coupling interface can result in higher vibration transmission at lower excitation frequencies. It is shown that the interface with double-well potential stiffness nonlinearity may yield chaotic responses that can significantly affect vibration transmission as indicated by time-averaged transmitted power. It is also found that cubic damping nonlinearity may cause lower time-averaged transmitted and dissipated powers at the interface in the vicinity of resonant frequency. These findings provide better understanding of the effects of nonlinearity at the interface on vibration transmission, and facilitate better designs of coupling interface for control of vibration transmission.

1. Introduction

The vibration power flow analysis (PFA) approach has become a widely accepted tool to characterise the dynamic behaviour of complex systems and coupled structures. Comparing with individual measures of vibration transmission using force and/or displacement transmissibilities, vibration power flow combines the effects of force and velocity amplitudes as well as their phase angle in a single quantity, and thus provides a better indicator of vibration transmission between various sub-systems of an integrated structure [1,2]. Suppression of vibration transmission may not be guaranteed by purely reducing force or displacement transmissibility. However, effective vibration mitigation can be expected when there is lower vibration energy transmission. Since the introduction of the fundamental concepts of power flows in ref. [1], many PFA techniques, such as a dynamic stiffness method [3], a mobility method [4], a travelling wave and scattering method [5], a power flow progressive approach [6], and an energy flow finite element approach [7] have been proposed and developed for linear dynamical systems such as beams, plates, plate-like structures and cylindrical shell structures. Xing and Price [8] proposed a more general PFA method based on the principle of continuum mechanics, in which power flow density vector was defined. The application of this method was demonstrated through analysis of some simple structures. However, solutions to the power flow equations of complex structures may be challenging and need further study. Xiong et al. [9] provided a general linear mathematical framework for PFA of structural-control systems. Both mobility-based and damping-based power flow mode theories were also proposed, which may be used to determine vibration power transmission in linear dynamical systems and also to achieve specific power flow patterns by modifying damping distribution within a dynamical system [10,11].

Although much research has been carried out to understand the power flow behaviour of linear vibration systems, there has been limited work reported on power flow characteristics of nonlinear dynamical systems. However, engineering structures are inherently nonlinear in nature and they may exhibit strong nonlinear phenomena, such as

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https://doi.org/10.1016/j.ijmecsci.2018.01.014
Received 19 June 2017; Accepted 10 January 2018
Available online 31 January 2018
0020-7403/

limit cycle oscillations, bifurcation and chaos. There is also an increasing use of lightweight, flexible structures which may undergo large deformation and function in the nonlinear plastic regime. The associated power flow behaviour remains largely unclear. There is also a growing interest in exploiting nonlinearity in the design of nonlinear vibration mitigation systems to enhance vibration attenuation [12]. One example is the use of geometric nonlinearities of some mechanisms to create nonlinear negative stiffness, which when used in combination with linear supporting stiffness, can lead to a low linearised natural frequency and thus provide benefits for vibration isolation [13]. While force and displacement transmissibilities have been used to evaluate the performance of nonlinear vibration isolators [14], much less work has been reported on the use of vibration power flow as the performance indicator of nonlinear isolators [13]. It is worth noting that to reveal the power flow and vibration transmission characteristics of nonlinear systems, linear PFA theories cannot be directly applied. It is therefore necessary to develop effective power flow methods for nonlinear dynamical systems to examine their inherent power flow behaviour.

There have been growing interests in investigations of nonlinear dynamical systems from a vibration power and energy perspective. Royston and Singh [15] used power flow transmission as a performance index in the optimisation of a multiple degree-of-freedom nonlinear mounting system. The authors also investigated an automotive hydraulic engine mount and showed the vibration power flow from an excited rigid body through a nonlinear path to a resonant receiver [16]. Xiong et al. [17] investigated the vibration power flow behaviour of an interactive system consisting of a machine, a nonlinear isolator and a flexible ship excited by sea waves. The isolator was characterised by pth power damping and qth power stiffness and the input power was shown to be only locally sensitive to the nonlinearities. Vakakis et al. [18] investigated the time-averaged power flow of conservative nonlinear systems and revealed targeted energy transfer phenomenon, which corresponds to a one-way channelling of vibration energy from a primary structure to a nonlinear attachment. Yang et al. [19–21] investigated power flow characteristics of the Duffing oscillator [19], a nonlinear vibration isolation system with a negatives stiffness mechanism [13], the performance of a nonlinear dynamic vibration absorber [20] and also a 2DOF system with an excited machine mounted on a nonlinear base structure via a nonlinear isolator [21]. It was found that the time-averaged input power associated with a chaotic response of the Duffing oscillator tends to an asymptotic value as the averaging time increases. Also, the asymptotic value of this time-averaged input power was shown insensitive to the initial conditions of a single chaotic attractor [19]. Therefore, time-averaged input power may be employed to quantify vibration transmission level associated with periodic, aperiodic and chaotic responses. It can thus be used as a uniform index for direct comparisons of the level of vibration transmission within a nonlinear dynamical system exhibiting different types of possible responses.

It should be noted that systems with nonlinear coupling interface or nonlinear joints at the connection point are frequently encountered in engineering applications. For instance, aircraft engines often contain nonlinear joints such underplatform dampers, blade roots and flange joints [22]. Many built-up structures contain bolted joints with nonlinearity caused by slipping of contacting surfacing and opening and closure of interfacial gaps [23]. The interfacial nonlinearities in the structures may exert significant influence over the dynamic characteristics of the integrated system. There have been some studies reported on the dynamic behaviour of structures with local nonlinearities [24,25]. Nevertheless, the effects of nonlinearities on vibration transmission within nonlinear systems still need further investigations. The influence of nonlinearities on the vibration power flow through nonlinear coupling interfaces remains unclear. It is therefore of importance to develop vibration PFA methods to evaluate the influence of coupling nonlinearities on vibration transmission, to reveal vibration energy transfer characteristics, and to enhance dynamic design by exploiting nonlinearities.

In this paper, the vibration transmission between interactive oscillators with a nonlinear coupling interface is investigated from both a force transmission viewpoint and also from a vibration power flow perspective. The effects of stiffness nonlinearity with hardening, softening, or double-well potential characteristics and cubic damping nonlinearity on vibration transmission are examined. Both analytical approximation and numerical integration methods will be used to evaluate vibration power input, dissipation and transmission as well as force transmissibility. The remaining content of the paper is organised as follows. The mathematical model of the system and analytical derivations will be provided in Section 2. Section 3 deals with formulations of vibration power flow, kinetic energies and force transmissibility for evaluations of vibration transmission levels. Section 4 investigates the effects of the damping nonlinearity and different types of stiffness nonlinearities on the time-averaged vibration power flow and force transmission. Conclusions are drawn at the end of the paper.

2. Mathematical modelling

2.1. Model descriptions and the governing equations

Fig. 1 shows the system model comprising two subsystems with a nonlinear coupling interface. Subsystem one is a single degree-of-freedom (DOF) system consisting of a mass m_1 subject to a harmonic excitation of amplitude f and frequency ω, a viscous damper of damping coefficient c_1, and a linear spring with stiffness coefficient k_1. Subsystem two is another single DOF system having mass m_2, a viscous damper with damping coefficient c_2, and a linear spring with stiffness coefficient k_2. The masses both move horizontally without frictions and their static equilibrium positions, where $x_1 = x_2 = 0$ and the springs are un-stretched, are taking as the reference. The coupling interface between the two subsystems is characterised by nonlinearities in both damping and stiffness. The damping force and restoring force of the nonlinear damper and the nonlinear spring at the nonlinear interface are $G(\dot{\delta})$ and $H(\delta)$, respectively, which are expressed by

$$G(\dot{\delta}) = c_3\dot{\delta} + c_4\dot{\delta}^3, \tag{1a}$$

$$H(\delta) = k_3\delta + k_4\delta^3, \tag{1b}$$

where $\delta = x_2 - x_1$ is the relative displacement of the masses, c_3 and c_4 are constant linear and nonlinear damping coefficients, respectively, k_3 and k_4 are the constant linear and nonlinear stiffness coefficients, respectively.

The dynamic governing equations of the integrated system are

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 - G(\dot{\delta}) - H(\delta) = f\cos\omega t, \tag{2a}$$

$$m_2\ddot{x}_2 + c_2\dot{x}_2 + k_2x_2 + G(\dot{\delta}) + H(\delta) = 0. \tag{2b}$$

To facilitate later derivations, the following dimensionless parameters are introduced

$$\omega_1 = \sqrt{\frac{k_1}{m_1}},\quad \omega_2 = \sqrt{\frac{k_2}{m_2}},\quad \mu = \frac{m_2}{m_1},\quad \lambda = \frac{k_3}{k_1},$$
$$\beta = \frac{k_4 l_0^2}{k_1},\quad X_1 = \frac{x_1}{l_0},\quad X_2 = \frac{x_2}{l_0},\quad Y = X_2 - X_1,$$
$$\gamma = \frac{\omega_2}{\omega_1},\quad \zeta_1 = \frac{c_1}{2m_1\omega_1},\quad \zeta_2 = \frac{c_2}{2m_2\omega_2},\quad \rho = \frac{c_3}{c_1},$$
$$\epsilon = \frac{c_4\omega_1^2 l_0^2}{c_3},\quad F = \frac{f}{k_1 l_0},\quad \Omega = \frac{\omega}{\omega_1},\quad \tau = \omega_1 t,$$

where ω_1 and ω_2 are the undamped natural frequencies of subsystems one and two, respectively, μ is the mass ratio, λ and β are the linear and nonlinear stiffness ratios of the spring at the interface, respectively, γ is the natural frequency ratio, l_0 is assumed to be the un-stretched length of the spring on the left, X_1 and X_2 are the non-dimensional displacements of masses m_1 and m_2, respectively, Y is the non-dimensional