International Journal of Mechanical Sciences 136 (2018) 134–142

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

An origami-inspired structure with graded stiffness

Jiayao Ma[a,b,1], Jichao Song[b,1], Yan Chen[a,b,*]

[a] Key Laboratory of Mechanism and Equipment Design of Ministry of Education, Tianjin University, Tianjin 300072, China
[b] School of Mechanical Engineering, Tianjin University, Tianjin 300072, China

A R T I C L E I N F O

Keywords:
Rigid origami
Functionally graded structure
Self locking
Graded stiffness
Energy absorption

A B S T R A C T

Origami-inspired structures and mechanical metamaterials are often made up of individual tessellating repeat units, the folding and relative geometry of which determine the overall mechanical properties. If these units are identical, then the mechanical behaviour of the structure is uniform throughout, meaning that it is not able to adapt to changeable loading conditions. Here we create and study an origami structure, based on the Miura-ori folding pattern, which has a varying geometry over its volume and graded stiffness. Using kinematic analysis, we show how geometric parameters of the folding pattern can be varied to create both rigid foldable and self locking stages. We demonstrate both experimentally and numerically that the structure can achieve periodically graded stiffness when subjected to quasi-static out-of-plane compression, and the mechanical responses can be tuned by changing the underlying geometric design. We obtain a structure with superior energy absorption capability to uniform tessellating repeat units, and anticipate that this strategy could be extended to other structures and metamaterials to impart them with non-uniform and graded mechanical properties.

1. Introduction

Origami has recently seen a surge in research interests from mathematicians, engineers and physicists. Making use of the superior ability of origami to transform 2D sheet materials into complicated 3D shapes, researchers have developed novel structures with a wide range of engineering applications, such as deployable structures [1,2], medical foldable devices [3], energy absorption structures [4–11] and mechanical metamaterials [12–22].

When prefolding a thin-walled structure following a proper origami pattern, its buckling and post-buckling modes can be pre-designed, which leads to a device with low peak force and high energy absorption efficiency. Ma and You [4] applied a novel origami pattern to square tubes in order to induce a complete diamond mode with doubled number of travelling plastic hinges. Quasi-static numerical simulations showed that both low peak force and high energy absorption efficiency were achieved in comparison with those of conventional square tubes. Experimental studies on the dynamic performance [5] and imperfection sensitivity [6] of the origami crash box were also conducted. Kite-shape pattern [7] and Tachi-Miura polyhedron bellows [8] were also adopted in the design of energy absorption devices, and an improvement in energy absorption capacity was obtained. Regarding circular tubes, Yang et al. [9] applied diamond origami pattern in circular tubes and achieved a significantly reduced peak force while the energy absorption was not compromised. Wu et al. [10] studied sinusoidal corrugation tubes and found out that both the initial peak crushing force and fluctuations of force–displacement curve were mitigated by the pattern.

In addition, origami-inspired mechanical metamaterials have gained increasing popularity in the ongoing quest for materials with exotic properties such as negative Poisson's ratio [12–17] and programmable stiffness [18–20]. Schenk and Guest [12] were among the first to propose a Miura-ori [1] based metamaterial and investigated its geometry, density, and Poisson's ratio in details. Lv et al. [13] also studied the geometry of Miura-folded plates and reported that the Poisson's ratio could be positive by varying pattern geometry. Wei et al. [14] conducted a thorough theoretical analysis of the three-dimensional elastic response of a single layer of Miura-ori folded sheet. By stacking Miura-ori sheets, curved origami structures with great morphing capability and self locking response were created [23]. In addition, multi-stable elastic behaviours [24] and structural weight reduction [25] were also achieved. To tune the stiffness of Miura-folded sheets, Silverberg et al. [18] introduced a kind of pop-through defect and obtained the correlation between the in-plane compression stiffness and the number, location, and combination of the defect. Besides, Miura-derivative fold patterns [21] and square twist pattern [22] have also been investigated in the design of origami mechanical metamaterials.

Up to now, most origami structures and metamaterials are formed by repetition of identical pattern unit cells. Those units are either rigid

* Corresponding author at: School of Mechanical Engineering, Tianjin University, Tianjin 300072, China.
E-mail address: yan_chen@tju.edu.cn (Y. Chen).

[1] Joint first authors.

https://doi.org/10.1016/j.ijmecsci.2017.12.026
Received 14 November 2017; Received in revised form 13 December 2017; Accepted 17 December 2017
Available online 18 December 2017

foldable ones, in which the deformation is only caused by rotation of creases while the panels remain rigid, to take advantage of the mechanism motion during deformation [17], or non-rigid ones to involve global material bending [22]. For a given origami pattern, besides the material selected, the mechanical behaviour is mainly determined by its geometric parameters, and therefore identical unit cells usually lead to uniform structural stiffness. However, under certain circumstances, a graded stiffness could enhance the functionality of metamaterials. For instance, in the design of the body of a transport vehicle which is soft when in contact with a human and but can be very rigid to protect occupants in a crash [26,27], or non-lethal projectiles for crowd control or peace support operations [28], materials with a graded stiffness is advantageous over those with a uniform stiffness. Such functional graded mechanical properties have been widely seen in nature, e.g., bamboos and bones have specifically graded distribution of cell number or cell size to adapt to their expected service environments [29–31]. Man-made functionally graded structures and materials have also been achieved, mainly by means of changing material density [32,33] or thickness [34–36]. However, no origami structure, which is able to systematically achieve graded stiffness through a proper geometric design while still preserving the native folding behaviours of the origami pattern, has been reported. Compared to previous designs with varying density or thickness, such structure can be made out of a single type of sheet materials with a uniform thickness, thereby greatly reducing the manufacturing costs.

Therefore, the objective of this paper is to design an origami structure based on the Miura-ori pattern, which is constructed by unit cells that are geometrically varied over the volume of the material, in order to create graded stiffness and high energy absorption when subjected to quasi-static out-of-plane compression. The layout of the paper is as follows. The geometry of the graded origami structure is first presented in Section 2. Section 3 gives details about the experimental setup and finite element modelling approach that are adopted in the quasi-static compression study of the origami structure. Subsequently are the results and discussions in Section 4. And the conclusion is given in Section 5 which ends the paper.

2. Geometry

2.1. Geometric design

A unit cell of the Miura-ori pattern as shown in Fig. 1(a) is composed of four creases meeting at one point, dividing it into four identical parallelogram panels. It is parameterised by two side lengths a and b, sector angle ϕ, and folding angle θ. The edge angles η and γ, and the dimensions w, l, v, and h can be calculated from the four independent parameters through the following equations [37]

$$\cos\gamma = \frac{\sin^2\phi\cos^2(\theta/2) - \cos^2\phi}{\sin^2\phi\cos^2(\theta/2) + \cos^2\phi} \tag{1}$$

$$\cos\eta = \sin^2\phi\cos\theta + \cos^2\phi \tag{2}$$

$$w = 2b\cdot\sin(\eta/2) \tag{3}$$

$$h = a\cdot\cos(\gamma/2) \tag{4}$$

$$l = 2a\cdot\sin(\gamma/2) \tag{5}$$

$$v = b\cdot\cos(\eta/2) \tag{6}$$

Tessellating the unit cells in the in-plane x and y directions leads to a Miura-ori sheet, such as the ones with 2 by 2 unit cells in Fig. 1(b). To create a 3D structure with geometric gradient, sheets with different sector angles are stacked in the out-of-plane z direction. For instance, joining the two sheets in Fig. 1(b) leads to a two-layer graded structure. By this means, a graded structure with multiple layers can be formed provided that the following geometric constraints are satisfied [12]

$$a_j = \frac{a_1\cos\phi_1}{\cos\phi_j} \tag{7}$$

$$b_j = b_1 \tag{8}$$

$$\theta_j = \cos^{-1}\left(1 - \frac{2\sin^2(\theta_1/2)\sin^2\phi_1}{\sin^2\phi_j}\right) \tag{9}$$

Here a_1, b_1, ϕ_1, θ_1 and a_j, b_j, ϕ_j, θ_j are respectively the geometric parameters in the base layer which is always defined as the layer with the smallest sector angle and denoted as layer 1, and those in an arbitrary layer j ($j = 2,3,\ldots$). Therefore for a multi-layer structure, when the sector angles of all the layers, and the side lengths and folding angle of layer 1 are determined, the geometry of the structure is completely decided. And the overall width W, length L, and height H of the structure can also be obtained

$$W = 2mb_1\cdot\sin(\eta_1/2) \tag{10}$$

$$L = 2na_1\cdot\sin(\gamma_1/2) + b_1\cdot\cos(\eta_1/2) \tag{11}$$

$$H = \sum_{j=1}^{p} a_j\cdot\cos(\gamma_j/2) \tag{12}$$

in which m and n are respectively the numbers of unit cells in a single layer in the x and y directions and p is the number of layers stacked in the z direction. Obviously, if all the layers have identical sector angles, a graded structure reduces to a uniform structure with identical unit cells.

2.2. Self locking

It can be seen from Eq. (9) that due to the variation in sector angles, the folding angles of the layers are also different, and layer 1 with the smallest sector angle ϕ_1 has the largest folding angle θ_1. As a result, a graded structure is only rigid foldable, i.e., the deformation is only caused by rotation of creases while the panels remain rigid, within the range of $0 \le \theta_1 \le 180°$. This point is illustrated by the rigid folding process of the two-layer structure in Fig. 1(c). It can be seen that when $\theta_1 = 180°$, layer 1 is completely flattened whereas layer 2 is still partially folded, and the rigid folding motion of the structure is locked by layer 1, i.e., θ_2 in layer 2 cannot further increase to make this layer flat. After that, the structure has to deform its panels to be further unfolded. The dimensions of a structure at the self locking point can be calculated as follows

$$W_l = 2mb_1\cdot\sin\phi_1 \tag{13}$$

$$L_l = 2na_1 + b_1\cdot\cos\phi_1 \tag{14}$$

$$H_l = a_1\sum_{j=1}^{p}\frac{\sqrt{\sin^2\phi_j - \sin^2\phi_1}}{\cos\phi_j} \tag{15}$$

where the subscript l means locking. And the displacement d in the z direction from the original to the self locking configurations can also be worked out

$$d = H - H_l = a_1\sum_{j=1}^{p}\frac{\cos\phi_1\cos(\gamma_j/2) - \sqrt{\sin^2\phi_j - \sin^2\phi_1}}{\cos\phi_j} \tag{16}$$