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Nonlinear light-induced vibration behavior of liquid crystal elastomer beam

Ahmad Mahdian Parrany

Department of Mechanical Engineering, Vali-e-Asr University of Rafsanjan, Rafsanjan, Iran

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ABSTRACT

Recent studies have shown the coupling of optical and mechanical energy in a class of liquid crystal elastomers that contain light-sensitive molecules. As shown experimentally in the literature, these materials can undergo large, reversible elastic deformation under light illumination, and therefore geometric nonlinearity effects can be significant. In this paper, we present a large deflection model for the light-induced bending vibration of a liquid crystal elastomer beam. In this regard, the von Karman's nonlinear strain–displacement relationship is used to account for the large deflection of the beam. The effect of light on the liquid crystal elastomer beam is modeled as an inhomogeneous and time-dependent light-induced contraction strain and the dynamic equations of the beam are derived using the Hamilton's principle. Finite element formulation is developed to analyze the nonlinear dynamic response of the beam under uniform light illuminations. In addition, numerical results are presented and effects of different physical and geometrical parameters, including light intensity, light source position, contraction coefficient, and the thickness of the beam on the vibration characteristics of the beam are investigated. The model developed in this paper can be used to design liquid crystal elastomer based structures such as optically sensors, photo-mechanical energy harvesters, or other reversible opto-mechanically active structures.

1. Introduction

Liquid crystal elastomers (LCEs) are made from a combination of liquid crystal and polymer network. In other words, they are a kind of cross-linked liquid crystal polymers which combine the elasticity properties of a network of long molecular chains in polymeric elastomers with the orientational order of shorter and stiffer molecules exhibited by liquid crystals [1–4].

The most important property of these materials is their ability to change their shape reversibly in response to a variety of external stimuli. To give more details, it should be mentioned that the nematic mesogens of these materials are characterized by an average direction of alignment, called the director **n** [5], and an order parameter Q [1,4,6]. Because of the direct coupling between the orientational order (the rotation of liquid crystal molecules) and mechanical deformation (the deformation of LCEs), any source that changes the orientational order Q will result in shape change [1,4].

Previous research has shown that the order parameter Q can be manipulated, and subsequently LCEs can be stimulated by thermal [7–9], electrical [10,23], magnetic [11], or optical [1,12–17] forms of energy to generate different mechanical outputs. In addition, these materials are fast (on the timescale of 10^{-2} s) [15,18], reversible [12], and able to generate considerable strains [12,15,19]. This is the reason why LCEs have received considerable attention in many engineering applications since they were first reported in 1967 [14,15,20–23].

The above mentioned unique features of LCEs have made them interesting candidates for diverse applications, including actuators [14,20,21,24,25], motors [23], tunable optical elements [26–28], microfluidics [29], and artificial muscles [30–33].

Amongst the above mentioned stimuli, photo-induced actuation has been the most attractive because it does not require contact and can be remotely, conveniently, and precisely triggered potentially over long distances. It is worthy to mention that this important ability comes from the fact that light can be used to modulate the order parameter Q in the materials in which the liquid crystal molecules have a special molecular structure [1]. In order to achieve photo-actuation in LCEs, light-absorbing components or dyes sensitive to light such as carbon nanotubes (CNTs) [34,35], and azobenzene moieties (a widely used compound that produce macroscopic strains and deformation under optical irradiation) [14,15,17,36–38] must be incorporated into the polymer matrix [12,16].

It is an undeniable fact that the ability to control the intensity, polarization, placement, and duration of light irradiation distinguishes this novel input stimulus in mechanically active systems. So, due to this ability as well as the other unique features of LCEs, these materials are becoming a topic of great technological and academic interest [39,40]. The increasing number of papers published in this area confirms this idea.

The idea of converting light energy into mechanical work through a polymer matrix was first discussed by Lovrien in 1967 [41]. Lovrien used

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E-mail address: a.mahdian@vru.ac.ir

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the combination of photosensitive chromophores such as azobenzene in a polymer host to hit the target.

The coupling of the orientational order and mechanical strain was first studied theoretically by de Gennes in 1975 [42] and then verified experimentally by Finkelmann et al. in 1981 [43].

Despite the considerable effort that was subsequently followed, photo-driven contraction larger than 1% was not obtained until 2001 when Finkelmann et al. synthesized an LCE, with an azobenzene based molecule, which led to substantial photo-generated contractile strains (approximately 20%) [12]. So, it was demonstrated that light can induce a large contraction up to 20% if some photochromic liquid crystal molecules such as azobenzenes are added into the polymer matrix [12]. Also, over 500% strain has been reported for nematic elastomer fibers in certain conditions [1,44]. As a result, the realization of macroscopic large light-induced deformation made these materials more exciting and applicable.

The kinetics of thermally induced bending of a nematic cantilever was modeled by Hon et al. [45]. The bending kinetics of an LCE-CNT composite cantilever which was stimulated by irradiation of laser diode (LD) light was addressed in [35]. Additionally, temperature evolution inside the cantilever, and the relation of bending kinetics with the applied irradiation power were measured directly [35,46].

Several photomechanical models have been built up to study the photomechanical behavior of light-activated polymer systems [47,48]. The surface deformation of nematic elastomers, containing azobenzene chromophores, under the spot as well as striped illumination has been simulated in [49,50]. Meanwhile, the case of inhomogeneous illumination, thermal and swelling fields, in beams, plates, and films made of photoresponsive solids was considered using small deflection theory in [51]. Anisotropic bending and unbending behavior of azobenzene-containing liquid-crystalline gels and films has been investigated in [36].

Considering the geometric nonlinearity, Dunn et al. simulated the deformation of thin-film plates with anisotropic photostrains that arise from irradiation by light using finite element method [4]. The deformation of the monodomain and polydomain LCE films under uniform polarized light with considering the effect of geometric nonlinearity was also calculated in [52]. Besides, the anisotropic photoinduced bending of non-crystalline molecular films under the absorption of linearly and circularly polarized light with considering the geometric nonlinearity was simulated in [53].

Jin et al. presented a large deflection light-induced bending model for liquid crystal elastomers under uniform or non-uniform illumination. In their model, the in-plane membrane force and geometric nonlinearity were considered [54]. It was also shown that the deflection of the LCE sample can be varied and controlled by changing the illumination position, the illumination direction, the light intensity, and the distribution half width of the electric field [54].

In the background of vibrations and oscillations, light has been used to drive high frequency and large amplitude oscillations in structures made from a photosensitive liquid crystal polymer [55,56]. It was demonstrated that linearly polarized light can repeatedly and precisely bend films of a liquid-crystal network containing an azobenzene chromophore along any desired direction [14]. In experiments, stable photo-driven optical oscillators in the kHz range based on push–pull azophenols have been reported [57]. It was demonstrated that the oscillation frequency of these devices ranges within 2 and 10 kHz at room temperature and they show no signs of fatigue upon continuous work [57].

More recently, Li et al. investigated the light-driven bending vibration of a liquid crystal elastomer (LCE) beam [58]. It was demonstrated that the amplitude of the bending vibration of the LCE beam can be regulated by tuning light intensity, damping factor of the beam, and thermal relaxation time from *cis* to *trans* state, while the frequency of the vibration mainly depends on the thermal relaxation time [58].

As already mentioned, LCE samples are usually thin and large deformation in them may occur and as a result the effect of geometric nonlinearity is significant. Thus, developing a large deflection model with considering geometric nonlinearity seems essential. Although a few theoretical and experimental models have been developed to study large deflection of LCE structures, to the best of the author's knowledge, no theoretical study has been conducted to investigate the large deflection dynamics of LCE structures with considering the effect of geometric nonlinearity.

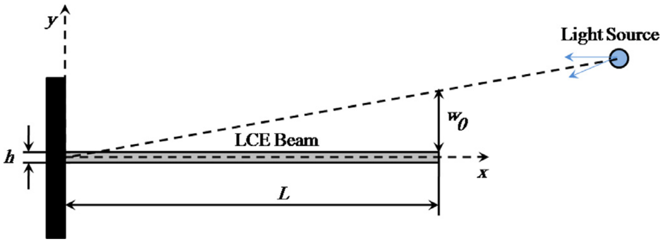

Fig. 1. Schematics of an LCE beam exposed to a static light source.

In LCE samples driven by light, the large amplitude and high frequency oscillations result from designing a novel geometry. Fig. 1 shows one of the most applicable geometries in this field. In this special type of geometry, cantilever beam made from photosensitive components is exposed to a uniform light source. In this system, when the upper surface of the cantilever is subjected to the light illumination, the beam bends upward due to the contraction of the upper surface of the cantilever. Then, after a while the lower surface of the cantilever is subjected to the light illumination and subsequently the beam bends downward. In other words, the light source drives oscillations by sequentially exposing the front and back surface of the cantilever beam. It has been proved experimentally that this system is able to vibrate steadily under a suitable light exposure [22].

These remotely accessed oscillations have potential application in a variety of areas. Specifically, these high frequency and large amplitude oscillations are used to utilize sunlight as the cheapest and most available source of energy. The system shown in Fig. 1 has been also proposed to develop solar energy harvesting devices [6,22].

In the present paper, the system shown in Fig. 1 is used to study the light-driven bending vibration of an LCE beam with taking account of geometric nonlinearity. As shown in Fig. 1, since the light source position is considered fixed, w_0 is defined based on the location of the light source and the cantilever beam. In addition, it is assumed that the upper surface of the beam is subjected to the uniform light illumination when the tip displacement of the beam is less than w_0, and the lower surface is subjected to the light illumination when the tip displacement of the beam is larger than w_0. In fact, the purpose of this work is to derive an efficient, high-fidelity, large deflection model for the light-driven bending vibration of a liquid crystal elastomer beam. The von Karman's nonlinear strain–displacement relationship is used to account for the geometrical nonlinearity caused by the large deflection of the beam. The governing differential equations of motion of the beam are derived using the Hamilton's principle, in which the effect of light on the liquid crystal elastomer beam is defined as an inhomogeneous and time-dependent light-induced contraction strain. Next, a finite element formulation is developed to analyze the nonlinear dynamic response of the beam under uniform light illuminations. In this regard, the Wilson-θ method is employed to determine the nonlinear dynamic response. Furthermore, the parametric studies are provided to examine the effects of physical and geometrical parameters, including light intensity, light source position, contraction coefficient, and the thickness of the beam on the nonlinear dynamic response of the beam.

This article is organized as follows. In Section 2, a brief summary of the photo-mechanical constitutive model for the LCE material is presented. The large deflection light-induced bending vibration model is derived using the Hamilton's principle in Section 3. In Section 4, a finite element formulation is carried out in order to analyze the nonlinear dynamic response of the LCE beam stimulated by light, and then results