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Plane strain cylindrical indentation of functionally graded half-plane with exponentially varying shear modulus in the presence of residual surface tension

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ABSTRACT

A functionally graded half-plane with shear modulus varying exponentially along the direction normal to the surface and surface effects accounted through Gurtin–Murdoch model, indented by long rigid smooth cylindrical indenter is solved to understand the effect of material inhomogeneity and surface effects on indentation response. The Green's function relating surface load to surface displacement under plane strain condition is obtained semi-analytically through the combination of Airy stress function approach and Fourier transforms and utilized to solve the contact problem. The solution is used to study the effect of inhomogeneity through grading parameter and surface effects through residual surface tension based intrinsic length scale on the contact pressure, contact size and in-plane normal stress on the surface responsible for cracks during indentation.

1. Introduction

Presence of material inhomogeneity in many naturally occurring material systems is claimed as the reason for their superior mechanical response [1]. Man-made efforts to exploit inhomogeneity with an aim to better mechanical performance as compared to their homogeneous counterparts has led to development of systems such as composites, layer-substrate (LS) systems, functionally graded (FG) materials. FG material, the focus of present study is a material system, where the material properties vary continuously and the variation is usually described through a mathematical function of position co-ordinates [2,3]. FG materials have garnered attention as the current material synthesis and processing capabilities can allow controlled gradations of materials over nanometer to microscopic length scales and based on it's functionality, it has found applications in areas as varied as bio-medical field, thermal barrier coating, sensors and ene.g. [4]. FG materials with varying co-efficient of thermal expansion are designed to withstand very high thermal gradient and used in space plane body, rocket engine component [5] and energy conversion devices like gas turbine engine as protective coating on turbine blades [6]. Artificial FG materials are used to replace damaged bones and teeth in human beings, which constitute examples of naturally found FG materials [7–9]. Tailor-made FG materials which resist against penetration and crack propagation have been proposed as the material for bullet-proof vest [10]. In the context of indentation problem, FG materials have shown better fracture resistance as compared to homogeneous material which suffer from Hertzian cone cracking [11–13]. The indentation problem with its many variation have also been explored simultaneously through modeling for FG material vis-a-vis homogeneous material and the following text attempts to summarize the most important works.

Bakirtas [14] was one of the first authors to solve the two-dimensional normal contact problem of rigid cylindrical and flat punch indenting a FG isotropic elastic half-plane with shear modulus varying exponentially along depth and constant Poisson's ratio using Airy stress function approach and Fourier integral transform and studied the effect of inhomogeneity parameter and Poisson's ratio on the stress distribution in the half-plane during indentation and in particular stress intensity factor in the flat punch case. Giannakopoulos and Suresh [15,16] carried out analytical and finite element (FE) studies of point load and axisymmetric indentation problem on a FG half-space with shear modulus varying as power law and exponential function of depth. It was shown that the contact response offered by the power law and exponential FG material varied significantly with operating conditions for instance, Poisson's ratio or rate of change of material stiffness along the depth direction. Suresh et al. [17], performed spherical indentation experiments on exponentially graded material and observed cracking (no cracking) on the surface depends upon the transition of stiffness as a function of depth from soft (stiff) to stiff (soft). The observation was rationalized through FE studies [17] which showed that the stiffness variation from soft (stiff) to stiff (soft) leads to tensile (compressive)

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principal stresses on the surface. Giannakopoulos and Pallot [18] proposed use of FG material with power law variation in shear modulus along depth for wear resistance sliding surfaces based on their study which revealed lesser principal tensile and shear stresses on surfaces under contact load in turn lessening the likelihood of fracture, plastic shake-down, grooving.

Ke and Wang [19,20] obtained the point load solution for multiple layers with linearly varying shear modulus firmly attached to a homogeneous substrate. The solution was used to approximate power law and exponential variation of shear modulus in a layer and further used to solve indentation problem with different indenter shapes. Ke and Wang extended their multi-layer approach to fretting contact [21], thermoelastic contact involving frictional heating [22] with an exponential variation in the shear modulus of layer attached to a homogeneous half-plane. Guler and Erdogan [23] solved the frictionless contact of rectangular and triangular rigid punch indenting an exponentially graded layer attached to a homogeneous half-plane. The work was extended to present a series of benchmarking solutions for the case of contact of two graded cylinders [24] and frictional sliding contact of parabolic and cylindrical rigid punch [25–27]. Chidlow et al. [28], solved the adhesive and non-adhesive contact problem of FG layer with exponential grading bonded to homogeneous half-plane using Airy stress function approach and Fourier series and noted that FG material also exhibit both JKR-like and DMT-like contact response. Unlike previous works, where the material properties varied in the direction of depth, Chen et al. [29,30], considered an arbitrarily oriented gradient half-plane with exponential grading and solved frictional contact problem using Fourier integral transform method and looked at the effect of gradient orientation angle apart from surface friction coefficient and non-homogeneity parameter on the contact pressure and in-plane surface stress. Comez [31] studied the contact problem for a rigid cylindrical punch moving steadily with a constant subsonic velocity on the surface of a functionally graded layer with constant Poisson's ratio and exponentially varying elastic modulus and mass density in the depth direction, bonded to a rigid substrate using Fourier integral transform technique and studied the effect of punch velocity on contact width and stress distribution in layer.

A lot many experiments have shown that the mechanical response of a variety of systems at sub-micron or nano level differs greatly from the systems at macro level because of the influence of surface effects on the system due to high surface to volume ratio [32]. One of the successful model to deal with surface effects is given by Gurtin and Murdoch [33,34], where the surface is treated as a mathematical layer of zero thickness and perfectly bonded with the bulk material [35]. The two parameters "residual surface tension" τ^s and "surface elasticity constant" κ^s governs the Gurtin-Murdoch (GM) model. The homogeneous half-plane or half-space considering surface effects, subjected to specified surface loading is studied [36,37], and the contact problem is solved [38–43] to obtain contact pressure and contact size. The results show that the loading length or contact size approaches the ratio of residual surface tension to elastic modulus known as surface intrinsic length scale parameter, the distribution of stresses, displacements profile, contact pressure and contact size shows significant difference in the presence and absence of surface effect. Inclusion of adhesion in the contact problem of homogeneous half-plane with residual surface tension have been carried out [44–47]. All these works consistently show that presence of surface tension along with adhesion leads to reduction in contact dimension and indentation depth compared to the classical JKR-limit and that the adhesion with surface effects acts a linking model connecting the limits of no surface tension (JKR-limit) and very high surface tension (Young-Laplace equation). Vasu and Bhandakkar [48] studied the cylindrical contact problem of LS system in the presence of surface effects and showed that the deviation of result from classical elasticity is more if the layer is stiff as compared to the substrate. Additionally a map defined by Hertzian contact size and layer to substrate shear modulus ratio was presented which defined regions where the response can be approximated by LS system in the presence and absence of surface effects and the layer as a homogeneous half-plane with or without considering surface effects. Finite element method has also been applied to solve the contact problem coupled with surface elasticity [42,49,50]. Attia and Mahmoud [49] carried out finite element simulations of the indentation of triangular, parabolic and flat punch on a FG layer with shear modulus varying as a power law function of depth bonded to homogeneous substrate considering GM surface elasticity model with second order displacement gradient contributions along the free surface and layer-substrate interface. Consistent with the studies on homogeneous materials, it is shown that for a given depth of penetration, the contact force required is more if surface elasticity is accounted in FG materials. Influence of surface elasticity on the contact pressure is more with increase in the gradation parameter leading to higher indentation force for the same indentation depth.

Thus it appears that barring the finite element based work of Attia and Mahmoud [49], no study is available which explores the combined effect of surface and functional grading on the indentation response in a semi-analytical framework, a problem the present work aims to address. Such study will clearly help to bring out the size effect of indenter on the indentation response in FG half-plane, an aspect which has been missing in the past and the recent works discussed above. Interestingly none of the works dealing with indentation on FG material quoted above explicitly provide contact size, an information equally valuable along with the contact pressure. The plan of the paper is as follows: Section 2 provides the description of the problem, necessary equations, boundary conditions in accordance with the GM model and the Green's function relating derivative of surface displacement to contact pressure obtained through Airy stress function approach and Fourier transform. The contact problem in terms of the unknown contact pressure and contact size is set up and the numerical procedure for it's solution is summarized in Appendix A. In Section 3, results are presented to analyze the impact of inhomogeneity and surface effects on the contact pressure, contact size and in-plane normal stress on the surface. Finally in Section 4, conclusions are presented.

2. Problem formulation

Fig. 1 shows the schematic of a Functionally Graded (FG) half-plane described by $x-y$ co-ordinates and indented quasi-statically by a long rigid and smooth cylindrical indenter of radius R symmetrical about x-axis. The resulting deformation of the half plane is assumed to be infinitesimal and obeying plane strain condition. The cylinder is subjected to a line load P leading to a contact over edge of length '$2a$' with 'a' as the semi-contact length. The FG half-plane is modeled as a linear elastic isotropic material with shear modulus $\mu=\mu(x)$ varying in the direction of indentation and a constant Poisson's ratio ν. The indenter and half-plane are assumed to be non-interacting so that the indentation is assumed to adhesion-free. The component of displacement along x and y direction in the half-plane are denoted by u_x and u_y respectively while the normal and shear component of stresses (strains) are represented as $\sigma_{aa}\,(\epsilon_{aa})$ and $\tau_{ab}\,(\gamma_{ab})$ respectively where a and b can denote x, y or z co-ordinates and $a\neq b$. The in-plane stress-strain relations under plane strain conditions are given by [51],

$$\epsilon_{xx}=\frac{1-\nu}{2\mu(x)}(\sigma_{xx}-\nu^*\sigma_{yy}),\ \epsilon_{yy}=\frac{1-\nu}{2\mu(x)}(\sigma_{yy}-\nu^*\sigma_{xx}),\ \gamma_{xy}=\frac{\tau_{xy}}{\mu(x)},\tag{1}$$

where $\nu^*=\nu/(1-\nu)$ and the compatibility condition binding the strains is [51],

$$\frac{\partial^2\epsilon_{xx}}{\partial y^2}+\frac{\partial^2\epsilon_{yy}}{\partial x^2}=\frac{\partial^2\gamma_{xy}}{\partial x\partial y}.\tag{2}$$

In the absence of body forces, the in-plane stresses in FG half-plane represented in terms of Airy stress function $\varphi(x, y)$ as,

$$\sigma_{xx}=\frac{\partial^2\varphi}{\partial y^2},\ \sigma_{yy}=\frac{\partial^2\varphi}{\partial x^2},\ \tau_{xy}=-\frac{\partial^2\varphi}{\partial x\partial y},\tag{3}$$