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Anisotropic complex diffusion filtering for comparison of the vortex identification methods in homogeneous turbulence

Waleed Abdel Kareem

Suez University, Faculty of Science, Department of Mathematics and Computer Science, Suez, Egypt

A R T I C L E I N F O

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A B S T R A C T

The vortex identification methods are mutually compared on the basis of extracting coherent vortices and isolating incoherent regions from a filtered turbulent velocity field. Those identification schemes are the high rotation Q−criterion, the second large eigen value λ_2−definition, the Δ−method, the vorticity magnitude $|\omega|$ and the new omega identification method ω_{new}. The velocity field of a forced homogeneous isotropic turbulent flow is obtained by simulating the lattice Boltzmann method (LBM) in a periodical box with resolutions of 128^3 and 256^3, respectively. The anisotropic diffusion model (ADM) is used to filter the velocity field into coherent and incoherent parts and finally the four identification methods are applied to identify the organized vortices as well as the noisy regions. The probability distribution functions (PDFs), the joint PDFs (JPDFs) and the spectrum of the extracted fields for the different identification methods are examined in details. Results show that the JPDFs have different shapes in the coherent cases, for example, the $(\lambda_2 - \Delta)$ and $(Q - \Delta)$-JPDFs for the non-filtered and coherent fields show an ellipse-like shape where the large axis is found in the Δ direction. The $(|\omega| - \Delta)$-JPDFs for the non-filtered and coherent fields show a half circle - like shape. The $(|\omega| - Q)$-JPDFs for the non - filtered and coherent fields inclined towards the positive direction of Q and it is longer at higher values of $|\omega|$. The $(|\omega| - \lambda_2)$-JPDFs for the non-filtered and coherent fields inclined towards the negative values of λ_2 and it is also longer for high values of $|\omega|$. The incoherent JPDFs for the Δ method with other identification methods are found constant and their maximum existence are mostly found for moderate values of Q and λ_2 at different values of Δ. However, the incoherent JPDFs $(|\omega| - Q)$ and $(|\omega| - \lambda_2)$ are almost similar and the incoherent JPDF for $(\lambda_2 - Q)$ is different, where most overlapping regions are found at negative values of Q and positive values of λ_2. There is a tendency to have a universal JPDFs between the different identification methods where the same JPDF shapes can be observed in the 128^3 and 256^3 cases.

1. Introduction

Many research efforts have been introduced for both filtering turbulent flow data and comparison between the vortex identification methods but the connection between the filtering methods and comparison of different identification methods is not addressed. Thresholding the flow data can not be effective in separating the incoherent flow from the raw data. Thus, there is a need to a filtering method to do the work. Some filtering techniques depend on the partial differential equations such as the level set method (Malladi et al. [1] and Sethian et al. [2]) and nonlinear diffusion (Guichard et al. [3], Barbu et al. [4]and Kareem et al. [5], [6]). There are also various methods for image filtering such as curvelets (Stark at al. [7], Tessens [8] and Herrmann [9]) and total variation method (Jia et al. [10] and Chen et al. [11]). Such filtering models have been also used for turbulent flow problems [12]. The main idea in filtering turbulent flow fields is to decompose the flow domain into two parts, namely the coherent and incoherent flows, respectively [13]. It is common to use the wavelet filtering analysis for turbulent flow fields ([14–16] and [17]). In turbulent flow studies, the filtering models are commonly applied against the velocity or vorticity vector fields. In this study, the complex anisotropic diffusion model (ADM) is applied to the velocity filed to decompose the velocity into coherent and incoherent parts [18]. Then the conventional vortex-identifying methods; Q [19], λ_2 [20], Δ [21] and the vorticity magnitude $|\omega|$ ([22], [23] and [24]); are used to identify the non-filtered field, the coherent and the incoherent fields.

The vortices in homogeneous isotropic turbulent flow are ideal for applying the vortex identification methods. It is known that the homogeneous isotropic turbulence is full by tube-like or worm-like vortices with different scales(Vincent and Meneguzzi [26], Kaneda et al. [27]). The simulation of forced turbulence can be performed using the standard spectral method to solve the Navier–Stokes equations or using the Lattice Boltzmann method (LBM) (cate et al. [28] and Kareem et al. [5]). These studies suggest that the energy spectrum in the inertial sub-range

E-mail address: walid.abdelkarim@suezuniv.edu.eg

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almost follows the $k^{-5/3}$ Kolmogorov scaling law [29], where k is the wave number. Visualization of the flow fields using one of the identification methods has been used widely in many 2D or 3D turbulent problems. The four identification definitions of vortices have been applied by Jeong and Hussain [20] to three numerically simulated flows. In the first flow, the tanh-profile temporal mixing layer that excited by spanwise perturbations is studied and the four definitions are compared. In the second problem, they considered the collision of two vortex rings and in the final example, the evolution of an axisymmetric column vortex was considered. Also, The comparison and relationships between the vortex identification methods Q, λ_2 and Δ for homogeneous isotropic turbulent flow are discussed extensively by Chakraborty et al. [30], where they compared the definitions against their new proposed identification method at zero and non-zero threshold values. They observed that all the five criteria result in remarkably similar geometrical vortical structures and they introduced a unified interpretation based on local flow kinematics for similarity or differences in the vortical structures. So, a comparison between the different identification criteria based on the anisotropic filtering is important and can support the theoretical studies about the differences and similarities between the identification criteria. The comparison can also be used to deduce some universal characteristics between the identification criteria in isotropic turbulence.

This paper is organized as follows. Section 2 discusses briefly the lattice Boltzmann method that has been used in the simulation of the turbulent field with a resolution of 128^3. The vortex mathematical definitions and their characteristics are discussed in Section 3. In Section 4., the filtering anisotropic diffusion model (ADM) is considered. The results of the study and the extensive comparisons between the four identification methods are carried out in Section 5. Section 6 is summarizing our conclusions.

2. Homogenous isotropic turbulent field

The lattice Boltzmann $D3Q19$ model (LBM) with a forcing term was used to simulate a 3D homogeneous isotropic turbulence in a periodic box of length $L_c = 128^3$ and the forced $D3Q15$ model (LBM) is used for the 256^3 case. It was shown in previous research efforts that the LBM can be derived from the Navier–Stokes equation (Chen et al. [31]). Also, The advances of the LBM in may fluid dynamics problems were discussed by Ubertini et al. [32] and Qi [33].

In this study, the Bhatnager–Gross-Krook (BGK) [34] single relaxation time approximation was used for the collision term and the lattice Boltzmann equation can be written as

$$\begin{aligned} &f_\alpha(\boldsymbol{x}+\boldsymbol{e}_\alpha\delta t, t+\delta t) - f_\alpha(\boldsymbol{x},t) \\ &= \frac{-1}{\tau}(f_\alpha(\boldsymbol{x},t) - f_\alpha^{eq}(\boldsymbol{x},t)) + 3\rho w_\alpha(\boldsymbol{e}_\alpha \cdot \boldsymbol{F}), \end{aligned} \tag{1}$$

The single relaxation time (SRT) is τ and the term $3\rho w_\alpha(\boldsymbol{e}_\alpha \cdot \boldsymbol{F})$ is the additional force to the Boltzmann equation. There are many ways to add force to the lattice Boltzmann equation such as shifting the velocity field or adding the force to the collision term, for example Luo [36], Shan and Chen [37], Guo et al. [38], Succi [35], Cosgrove at al. [39], Siggia and Patterson [40] and Mohseni et al. [41]. It was shown that such implementation of the force satisfies the continuity and Navier–Stokes equations up to the second order. The three components of the forcing function $\mathbf{F}$ which is used in this study can be written as

$$\mathbf{F} \equiv \begin{cases} F_x = a\left(\frac{k_2k_3}{k^2}\right)\sin\left(\frac{2\pi}{N}(k_1x+k_2y+k_3z)+\phi\right) \\ F_y = -2a\left(\frac{k_1k_3}{k^2}\right)\sin\left(\frac{2\pi}{N}(k_1x+k_2y+k_3z)+\phi\right), \\ F_z = a\left(\frac{k_1k_2}{k^2}\right)\sin\left(\frac{2\pi}{N}(k_1x+k_2y+k_3z)+\phi\right) \end{cases}$$

where ϕ is the random phase and a is the forcing amplitude [42] and it is chosen as 10^{-4} for both simulations. The force is injected at low cutoff wave-number which is chosen as $(0 <|\mathbf{k}|\le 3)$ where $|\mathbf{k}| = \sqrt{k_1^2+k_2^2+k_3^2}$ and the parameter N is chosen as $N = 128$ or $N = 256$ for the 128^3 and 256^3 simulations, respectively. Recently, this forcing method is used in studying the thermodynamics of a real fluid near the critical point in numerical simulations of isotropic turbulence [43] and in investigating a single droplet undergoing phase change in isotropic turbulence using numerical simulations with a hybrid lattice Boltzmann scheme [44]. The relaxation time is chosen as $\tau = 0.503$ and the lattice Boltzmann Method can simulate the Navier–Stokes equations with the viscosity $\nu = \frac{1}{3}(\tau - 0.5)\delta x$.

The equilibrium distribution function $f_\alpha^{eq}(\boldsymbol{x},t)$ at the equilibrium state can be written as

$$f_\alpha^{eq}(\boldsymbol{x},t) = w_\alpha\rho[1 + 3(\boldsymbol{e}_\alpha.\boldsymbol{u}) + \frac{9}{2}(\boldsymbol{e}_\alpha.\boldsymbol{u})^2 - \frac{3}{2}(\boldsymbol{u}.\boldsymbol{u})], \tag{2}$$

where the fluid mass density is ρ and the macroscopic fluid velocity is $\boldsymbol{u}$. The discrete velocity set for the $D3Q19$ model is $\boldsymbol{e}_\alpha$ and w_α represents the weighting coefficients that depend on $\boldsymbol{e}_\alpha$ and the dimensions of space. The discrete velocity set is defined by

$$\boldsymbol{e}_\alpha = \begin{cases} (0,0,0), & \alpha = 0 \\ (\pm1,0,0),(0,\pm1,0),(0,0,\pm1), & \alpha = 1-6 \\ (\pm1,\pm1,0),(\pm1,0,\pm1),(0,\pm1,\pm1), & \alpha = 7-18 \end{cases}.$$

The values of the weighting coefficients w_α are

$$w_\alpha = \begin{cases} \frac{1}{3}, & \alpha = 0 \\ \frac{1}{18}, & \alpha = 1-6 \\ \frac{1}{36}, & \alpha = 7-18 \end{cases}.$$

The macroscopic quantities including density and momentum can be obtained from the moments of the mesoscopic distribution function f_α as $\rho = \sum_\alpha f_\alpha$ and $\rho\mathbf{u} = \sum_\alpha f_\alpha \mathbf{e}_\alpha$, where $\mathbf{u}$ is the macroscopic velocity. One of the important results that is derived from the macroscopic velocity is the study of the energy spectrum function $E(k)$ behavior at the scalar wave number $k \equiv (\mathbf{k}\cdot\mathbf{k})^{\frac{1}{2}}$. The definition in spectral space is always written as $E(k) = \frac{1}{2}\sum_{k-\frac{1}{2}<|k|\le k+\frac{1}{2}} |\hat{u}^2(k)|$, where $\hat{u}$ represents the Fourier modes of the obtained velocity field.

3. Anisotropic diffusion model

Mahmoodi [45] investigated a complex anisotropic diffusion model (ADM) that demonstrated a high performance for noisy signals in comparison to the common Perona and Malik algorithm [46] to remove noisy parts from one dimensional signals. The complex anisotropic diffusion model that proposed by Mahmoodi for noise removal from image processing signals is generalized to a 3D model to identify the coherent and incoherent parts of the velocity field that generated by the LBM. The model is succeeded in previous studies (Kareem et al.([6] and [18])) to extract the coherent and incoherent flows of isotropic turbulence by applying the model against the scalar quantity Q without any decomposition of the turbulent scales [6] and against the velocity components of different scales [18]. The model developed in this study has superior performance over the other traditional diffusion models such as Perona and Malik [46] and other studies such as [47] where they studied the models to filter forced and decaying turbulence data that are simulated by Kareen et al. [42] and [48], respectively . The filtering in those previous efforts can be observed as low-pass filter, however in this model it is a band-pass filter. They remove the high frequency components of the original band-pass signal because it behaves like a low-pass filter. However the model here removes noise and also preserves discontinuities at high frequency wavenumbers. The results are obtained by assuming that the relation between the low-pass and band-pass signals are $h(\vec{r},t) = u(\vec{r},t)e^{i\vec{\beta}.\vec{r}}$ and similar results can be obtained by assuming $h(\vec{r},t) = u(\vec{r},t)e^{-i\vec{\beta}.\vec{r}}$ (The different parameters in these mathematical relations will be explained in details below). Here the model is applied