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Sensitivity analysis of parametric errors on the performance of a torsion quasi-zero-stiffness vibration isolator

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ABSTRACT

A quasi-zero-stiffness (QZS) isolator can perform a good function of low-frequency vibration isolation, but its designed ultra-low dynamic stiffness could be notably affected by parametric errors, which usually occur in the manufacturing and assembling process. In this paper, the effects of design parameter errors on the performance of a torsion QZS vibration isolator are studied, and several useful guidelines for the manufacture and assembly of the QZS isolator are achieved. An approximate relationship between a single error and the jump-down frequency is obtained, and then the influence of a combination of multiple errors is investigated numerically. The results indicate that, as key parts of the QZS isolator, the universal wheel and cam should avoid positive manufacturing tolerances, while the shaft connector has to keep away from negative tolerance, and an assembly error of the disk spring stack should be mostly avoided as much as possible. Additionally, the degradations on the performance caused by single errors can be eliminated by a combination of multiple errors, due to a mutual counteraction of these influences. Most importantly, the QZS vibration isolation system still outperforms its linear counterpart, even though it has parametric errors.

1. Introduction

A negative-stiffness (NS) mechanism combining in parallel with a positive-stiffness elastic element can form a quasi-zero stiffness (QZS) vibration isolator, which enables a function of low-frequency vibration isolation [1]. In past decades, many works have been done to build novel NS mechanisms by different mechanisms or structures, such as oblique spring [2–6], buckling beams [7,8], thin strip [9], magnetic spring [10–12], cam-roller-spring mechanism [13], disk spring [14], limb-like structure [15], scissor-like structure [16] and X-shaped structure [17]. By theoretical and experimental studies, it is found that the QZS vibration isolator has an excellent performance of low-frequency vibration isolation, comparing with the traditional linear isolator. Moreover, the QZS vibration isolation method has been extended from single-degree-of-freedom (DOF) to multi-DOF and multi-stage vibration isolation [18–25], which also show high effectiveness of low-frequency vibration isolation in multiple directions. However, the designed ultra-low dynamic stiffness around the static equilibrium position would be affected notably by parametric errors, which usually happens in the manufacturing and assembling process, leading to performance degradations, and even a failure of the vibration isolation system.

In general, parametric errors mainly include manufacturing tolerances and assembly errors, which would seriously destroy the accuracy and efficiency of a machine or a tool, such as permanent synchronous machine [26], induction motor [27], regenerative exchanger [28], double-ridged horn antenna [29], bearing [30] and machine tool [31]. For QZS vibration isolation systems, Huang et al. [32,33] analyzed the effects of the stiffness and load imperfections of a QZS isolator consisting of buckled Euler beams. Zhou et al. [34] also investigated the mismatch of designed payload. Both results indicate that the isolation performance of a QZS isolator significantly deteriorates if a loading mismatch occurs. However, the studies about the impacts of manufacturing tolerances and assembly errors on the performance of the QZS vibration isolation system are rarely reported.

In practice, the reason for the poor isolation performance of a QZS vibration isolation system is that the static equilibrium position deviates from the originally designed one and the dynamic stiffness drastically increased as a mismatch of payload happens. Nevertheless, the factors that cause deviation of the static equilibrium position are far more than the mismatch of payload. The manufacturing tolerances and assembly errors of a QZS isolation system also could result in the derivation of the static equilibrium position. Additionally, these errors would cause other negative effects on its performance.

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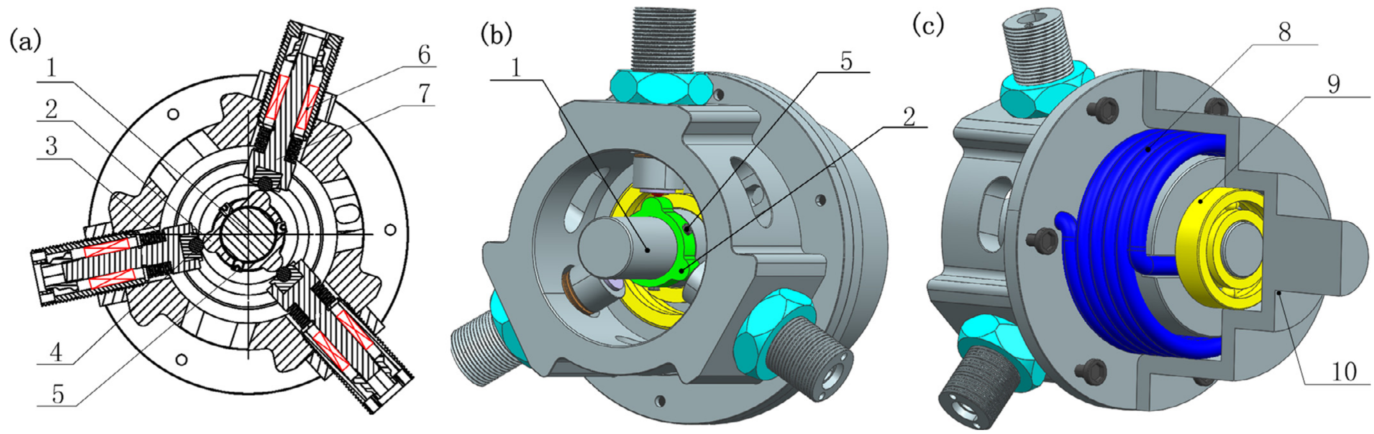

Fig. 1. Schematic diagram of the torsion QZS vibration isolator. (a) Transverse cross-sectional view, (b) left axonometric drawing, and (c) right axonometric drawing of the conceptual model. 1-left shaft connector, 2-cam with radius r_2, 3-disk spring stack, 4-universal wheel with radius r_1, 5-fastening screw, 6-linear bearing, 7-sliding rod, 8-torsion spring, 9-support bearing, 10-right shaft connector.

In order to study the influence of the manufacturing tolerances and assembly errors on the performance of a QZS vibration isolation system, this paper takes a torsion QZS isolator as an example. The conceptual model of the torsion QZS isolator was proposed in our previous work [34], but the detailed design solution was not given. For example, the special property of the rubber [35] would make the performance of the isolator uncertain, and a large size of the rubber coupling would be needed to fulfill QZS feature. In this paper, the torsion QZS isolator is developed from the original one [34] by using three pairs of cam and universal wheel supported by disk springs to construct a NS mechanism, and using a torsion coil spring to provide positive torsion stiffness.

The static and dynamic analyses of the torsion QZS isolator with parametric errors are carried out, and the performance of vibration isolation is evaluated in terms of torque transmissibility. The effects of single design parameter error, such as the manufacturing tolerance of each part and assembly error, on the performance of the QZS isolator are studied thoroughly. More importantly, the effects of multiple errors on the performance will also be studied in detail. This study would be a useful guideline for the manufacture and assembly of a torsion QZS isolator, which is also the original contribution of this paper.

This paper is organized as follows: In Section 2, the conceptual model of the torsion QZS isolator is presented and the static analysis is conducted. The torque transmissibility is given in Section 3 to evaluate vibration isolation performance. In Section 4, the effects of design parameter errors on the performance of the isolator are analyzed, and several useful guidelines are achieved for the manufacture and assembly of the isolator. Finally, some conclusions are drawn.

2. Statement of the model and static analysis

2.1. Conceptual model of the torsion QZS isolator

The schematic diagram of the torsion QZS isolator is shown in Fig. 1. Both ends of the isolator are connected with shafts, not only to transmit a designed torque M_0 but also to isolate the torsion oscillation from one shaft to the other one. The right-hand shaft connector (10) and the left-hand one (1) are elastically connected through a linear torsion spring (8) with positive stiffness k_θ (N · m/rad), which plays a role of shaft coupling. Three cylindrical cams (2) are symmetrically located on a sleeve that is fixed on the left shaft connector (1) by fastening screws (5). Three universal wheels with radius r_1 are fixed on three rods (7) accordingly. Such a rod (7) is guided by a linear bearing (6), and thus can only slide along the radial direction. All of the rods and the linear bearing are installed on the right-hand shaft connector. These three rollers contact with three cams correspondingly. It should be noted that the number of the cylindrical cam can be selected as $N(N \geq 2)$, which is dependent on the geometrical dimensions of the parts and the design requirement. Due to the restrictive size of the isolator, $N = 3$ is chosen for this conceptual design. Disk springs (3) are stacked upon the slide rod to provide a restoring force F_s when the universal wheel contact against the cam.

Table 1
The physical parameters of the isolator.

Physical parameters	Symbol	Unit
Pre-compression of disk spring stack	λ	(m)
Stiffness of torsion spring	k_θ	$(N\,m/rad)$
Radius of the universal wheel	r_1	(m)
Radius of the cam	r_2	(m)
Radius of the left shaft connector	r_3	(m)
Length of the roller outside of universal wheel	h	(m)
Inner radius of the disk spring	d	(m)
Outer radius of the disk spring	D	(m)
Thickness of the disk spring	t	(m)
Free height	h_0	(m)
Modulus of elasticity	E	(Pa)
Poisson's ratio	u	null
Piece of disk spring	n	piece

The friction between the cylindrical cam (2) and universal wheel (4) is neglected, because the rolling friction is much smaller than the sliding friction. In this paper, three pairs of cylindrical cam and universal wheel are constructed to realize torsion negative stiffness, and thus to counteract the positive stiffness of the torsion coil spring. When the negative stiffness is equal to the positive one at the static equilibrium position, the QZS property is achieved. Note that, at such a static equilibrium position, the centers of the cylindrical cam, the universal wheel and the shaft locate on the same straight line, as shown in Fig. 1a, and the pre-compressed deflection of the disk spring stack is denoted as λ. The static torsion angle of the left shaft relative to the right one is $\theta_0 = M_0/k_\theta$. The parameters of the QZS isolator are summarized in Table 1.

2.2. Static analysis

The schematic diagram of static analysis is depicted in Fig. 2. When the isolator is at rest, namely no loading torque applied on the isolator, as shown in Fig. 2a, the positive stiffness causes a rotational offset $\theta = -\theta_0$ from the static equilibrium position (Fig. 2b). As a payload M_0 is applied on the isolator quasi-statically, the angle θ increases slowly until the system reaches its static equilibrium state. At such a position, the centers of the universal wheel, cam and left-hand shaft will be on the same straight line, and thus the torques resulted from the disk