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# Bending collapse of multi-cell tubes

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ABSTRACT

Due to excellent energy absorption efficiency under bending collapse, multi-cell beams are extensively used as the components of protection structures. However, the theoretical prediction of their responses is still an unsolved problem. In this paper, two methods: dimensionless analysis method and energy analysis method are employed to derive the bending moment response of multi-cell tubes. Numerical simulations of double-cell tubes with different section dimensions are conducted first. The dimensionless analysis method is then employed to correlate the bending moment of double-cell tubes with the ratio of width to thickness $b/t$, the flow stress and the bending rotation angle. Based on Kecman's model, a theoretical model of double-cell tubes is presented and the bending moment response is derived according to the energy equilibrium of the system. Finally, the expression of a rolling radius of plastic hinge lines in the energy analysis method is determined to bridge the two methods. A comparison shows that the predictions provided by the present two methods agree well with the numerical results. Quadruple-cell tubes are also employed to validate the present methods and results show that the present methods are applicable for multi-cell tubes with other cross-sections.



## 1. Introduction

As high-efficiency energy absorbers, thin-walled structures have been widely used in cars, aircrafts, guardrails and so on. Nevertheless, over the last few decades, many attempts have been made to further improve the energy absorption efficiency of the thin-walled tubes under different loading conditions [1]. Foam-fillers are used to increase the energy absorption efficiency of the thin-walled tubes under axial crushing [2-6] and lateral impact [7-10]. Introducing graded wall thickness is another way and was validated to be very efficient in improving the specific energy absorption of thin-walled beams [11-16].

Recently, adopting multi-cell sections was found to be an effective way of improving structural crashworthiness and it was demonstrated by many numerical [17,18] and experimental studies [19-22]. In axial crushing experiments [19], the number of folding lobes is increased with the increase of cell number and the energy absorption efficiency of multi-cell tubes is at least 40% higher than that of single-cell tubes. Meanwhile, lots of theoretical studies [19,20] on the axial crush of multi-cell square or circular tubes were published, which could well predict the mean crushing force and energy absorption of multi-cell tubes. However, comparatively speaking, the studies on the bending collapse of multi-cell tubes are quite few and primarily focus on numerical or experimental aspect. Even no theoretical study on bending collapse of multi-cell tubes is found in the open literature due to the complexity of the problem.

A comprehensive study on the bending collapse of rectangular tubes was conducted by Kecman [23]. A theoretical model of empty rectangular tubes was established, and the energy was dissipated by the stationary and traveling hinge lines. Some parameters like the folding wavelength and the rolling radius were determined from experiment. Later, Wierzbicki et al. [24,25] developed a super folding element model, which was first used in axial crushing and then extended to the case of bending collapse. Unfortunately, the theory is not applicable for the pure bending with large deflection. In 2001, Kim and Reid [26] presented a self-consistent method to predict the response of the bending moment for the empty tube. However, the iterative process was required to solve this problem, and it is hence not convenient for applications. Many experiments have been carried out to investigate the bending collapse of empty tubes. Kecman [23] tested a serial of empty tubes by bending of cantilever beams to simulate the status of pure bending and showed a good agreement with his theory. Cimpoeru and Murry [27] performed four-point bending experiments by a testing machine designed to evaluate the moment-rotation properties of thin-walled tubes directly. Santosa [28] carried out three-point bending tests and found that the deformation pattern of the tensile flange is different for empty tubes under pure bending and three-point bending. In 2005, based on dimensionless

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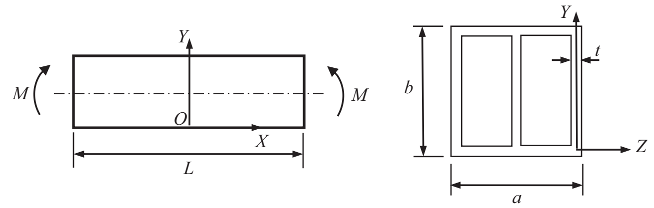


**Fig. 1.** Schematics of double-cell specimens under pure bending.

analysis method, Huang and Lu [29] predicted the bending moment response of empty square tubes and gave a theoretical expression of the bending moment, which agreed well with experiment [23].

More recently, Yin et al. [30] and Wang et al. [31] investigated the bending resistance of multi-cell tubes under three-point bending loads by numerical simulations. They both found that multi-cell tube outperformed single-cell tube in energy absorption efficiency. The influence of foam-filler on energy absorption of multi-cell tubes was also analyzed by Yin et al. [30]. Furthermore, Wang et al. [31] reported that double-cell tubes showed higher energy absorption efficiency than single-cell, triple-cell, and quadruple-cell tubes with the same mass under bending collapse. In general, multi-cell tubes showed much better performance than single-cell tubes under bending collapse.

In the present work, theoretical analysis of multi-cell tubes under pure bending is investigated. Numerical simulations are conducted first to obtain the bending responses of multi-cell tubes by using the non-linear finite element code LS-DYNA. Two methods: dimensionless analysis method and energy analysis method are then employed to predict the bending moment responses of multi-cell tubes. For dimensionless analysis method, square tubes are analyzed only for the sake of simplicity, while rectangular tubes are considered by energy analysis method. The relation between the two methods is analyzed, and the theoretical predictions are compared with simulation results for double-cell and quadruple-cell tubes.

## 2. Finite element analysis

As mentioned previously, the double-cell beams showed excellent bending resistance. In this section, a parametric study on the bending moment response of them under pure bending is carried out numerically. The double-cell section analyzed in this article is illustrated in Fig. 1. It is a rectangular double-cell section with the width $a$, height $b$, thickness $t$, and length $L = 70$ mm.

### 2.1. Analysis scheme

A number of square double-cell tubes with $a = b$ are analyzed first. Tubes with different width $b$ and wall thickness $t$ are analyzed with the aspect ratio $b/t$ ranging from 25.00 to 71.43. The specimens are classified into three groups: A, B, and C. For group A and B, the $b/t$ ratios of the specimens are kept constant to be 33.33 and 50.00, respectively. In group C, all the specimens have the same width $b$ but different thickness and hence different ratio $b/t$. The dimensions of all specimens in group A, B and C are listed in Table 1.

For rectangular double-cell tubes with $a \neq b$, a group of specimens D are considered. In group D, tubes with six different $a/b$ values: 0.5, 0.8, 1.0, 1.3, 1.6 and 2.0 are analyzed with a constant $b/t$ ratio of 33.33. The dimensions of these tubes are also given in Table 1.

### 2.2. Finite element modeling

Numerical simulations of pure bending collapse are carried out by using non-linear explicit finite element code LS-DYNA. The post-processor LS-PREPOST is used for data process and visualization. Mild

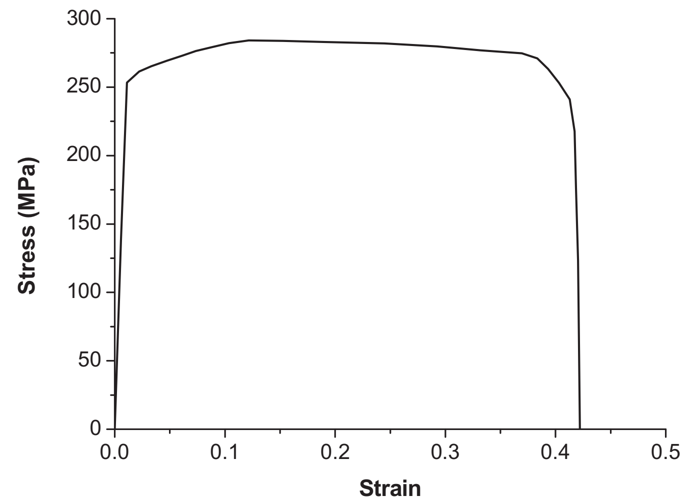


**Fig. 2.** Engineering stress–strain curve of mild steel in Kecman's [23] test.

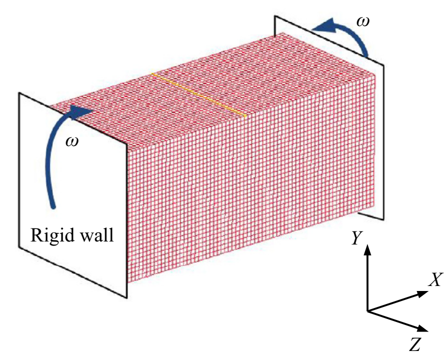


**Fig. 3.** The representative finite element models for square double-cell tubes.

steel adopted by Kecman [23] is employed here as the structural material of all tubes. The engineering stress–strain curve of the material is plotted in Fig. 2, and the mechanical parameters of it are given here: Young's modulus $E = 210$ GPa, initial yield strength $\sigma_y = 253$ MPa, the ultimate stress $\sigma_u = 284$ MPa and Poisson's ratio $\upsilon = 0.3$. The determination of flow stress is not easy, but in this analysis, it is simply defined as $\sigma_0 = \sigma_u$ since it is adopted by Kecman [23] and there is no significant difference between $\sigma_y$ and $\sigma_u$. The material type 24 (Piecewise linear plasticity material) provided by LS-DYNA is applied to simulate the material.

A representative finite element model for square double-cell tubes is present in Fig. 3. The double-cell tubes are modeled by using Belytschko–Tsay shell element with four nodes, and five integration points are used throughout the thickness. The characteristic mesh size of the tubes is 1.0 mm. Due to pure bending, collapse may occur in any position of the beam. A small indentation trigger is introduced at the top surface in the middle of the beam to guarantee stable bending collapse to initiate in the middle. The depth of the trigger is set to be proportional to the wall thickness with a coefficient of 5%. The nodes in the two ends of the beam are attached to two rigid plates which are rotated with constant angular velocity to assure pure bending of the beam. The two rigid plates are also modeled with Belytschko–Tsay shell element, and the angular velocity is set to $w = 0.5°$ per second. As shown in Fig. 3, the beam is rotated around the $Z$ axis. An automatic single surface contact algorithm is applied to consider the contact of the tube itself, and the friction coefficients are set as 0.3.