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Gaussian and non-Gaussian stochastic response of slender continua with time-varying length deployed in tall structures

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ABSTRACT

This paper presents a study to predict the probabilistic characteristics of lateral dynamic motions of a long heavy cable moving at speed within a tall host structure. The cable is subjected to a base-motion (kinematic) excitation due to a low frequency sway of the structure. The development of the deterministic equations of motion and of the stochastic models describing the lateral dynamic behaviour of the cable is presented. Due to the time-varying length of the cable, the system exhibits nonstationary dynamic characteristics and its response is governed by nonstationary ordinary differential equations. Two stochastic models of motion of the structure are considered. In the first model, the excitation is represented as a narrow-band Gaussian process mean-square equivalent to a harmonic process. The second model involves a non-Gaussian process in the form of a random train of pulses, idealizing the action of strong wind gusts. The differential equations to determine the mean values and the second-order joint statistical moments of the response are formulated and solved numerically. A parametric study is conducted to demonstrate the influence of speed of the cable on the deterministic and stochastic characteristics of the response.

1. Introduction

Environmental phenomena such as strong wind conditions and earthquakes cause tall civil structures such as towers and high-rise buildings to vibrate (sway) at low frequencies and large amplitudes [1,2]. When the structure is sufficiently flexible, the dynamic response to forces generated by these phenomena is significant. As a result, the corresponding kinematic excitation often excites long slender continua such as cables and ropes that are part of equipment hosted within the structure. For example, large resonance motions of suspension ropes and compensating cables in high-speed elevators in high-rise buildings take place [3]. In order to predict the dynamic behaviour of moving continua in such systems, various models have been used. The excitation mechanism can be represented by deterministic functions and consequently the response of the system is treated as a deterministic phenomenon [4–8]. However, the nature of loading caused by environmental phenomena such as wind is usually nondeterministic [9,10]. The excitation should then be described by a stochastic process so that the methods of stochastic dynamics can be employed to predict the dynamic behaviour of the system.

In this work, the model and stochastic methodology proposed by Kaczmarczyk et al. [11] is extended and used to carry out a comprehensive computer simulation study to predict the dynamic response of a long cable moving at speed within a tall slender host structure. First, a derivation of the deterministic model which describes the dynamic behaviour of the system is summarized and the dynamic characteristics of the system are explained. Two, stochastic models of motion of the structure are considered. In the first model, the excitation is represented as a narrow-band Gaussian process mean-square equivalent to a harmonic process. Alternatively, the dynamic loading due to wind gusts may be adequately idealized by a train of randomly occurring pulses [12–14] with the corresponding the dynamic response of a structure being also a train of pulses. Hence, in the second model, the excitation is treated as a non-Gaussian process in the form of a random train of pulses. For both models, the non-stationary differential equations governing the statistical moments of the state vector are presented. The

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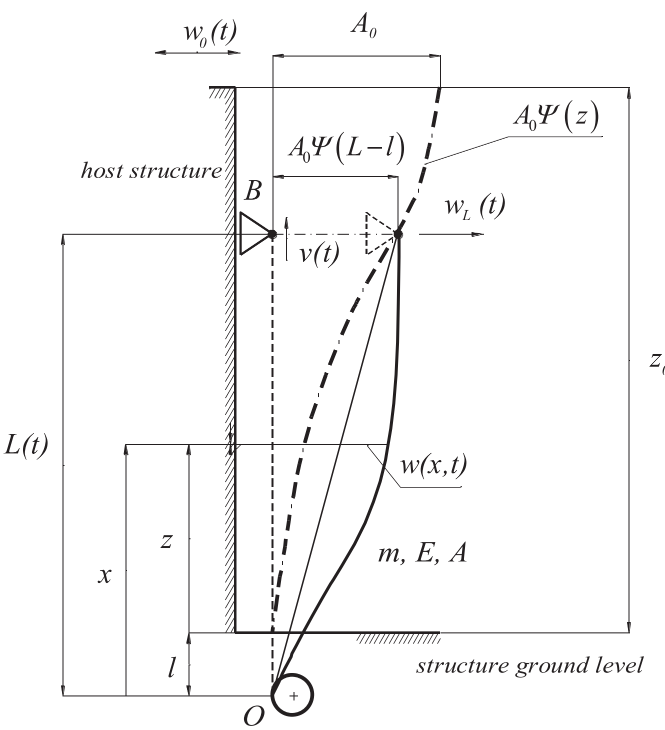

Fig. 1. Vertical cable moving within a tall structure.

equations are then solved numerically and a numerical study based on a range of model parameters of the system is conducted.

2. Equations of motion

The model depicted in Fig. 1 is used to study the dynamic behaviour of a vertical cable of time-varying length *L(t)*. The cable which has mass per unit length m is wrapped around a drum at the bottom end and attached at B to a support moving at speed v within a tall cantilevered host structure. The host structure sways which results in motion $w_0(t)$ of amplitude A_0 at the level defined by the coordinate z_0 measured from the structure base level. The deformations of the structure are described as $A_0\Psi(z)$ where $\Psi(z)$ represents the deformation shape function with z denoting a coordinate measured from the base level. The base motion $w_0(t)$ excites the cable and its dynamic response is represented by the lateral displacements denoted as *w(x,t)*, where x is measured from the origin O placed at distance l below the base level.

The equations of motion of the system presented in what follows are based on the model discussed in [11], with a more accurate representation of the deformations of the structure used. The mean quasi-static tension of the cable is expressed as

$$T_m(x,t) = T_0 + mx[g + a(t)] \tag{1}$$

where the spatial coordinate x is defined in a time-variant domain $0 < x < L(t)$, T_0 represents a constant tension term, $a(t) = \dot{v}(t)$ is the acceleration of the upper support (an overdot denotes the time derivative) and g is the acceleration of gravity.

The equation governing the linear undamped dynamic response of the cable in terms of the lateral displacements *w(x,t)* is given as

$$m\frac{\mathrm{d}^2 w}{\mathrm{d}t^2} - \left[T_0 + m(g+a)x\right]w_{xx} - m(g+a)w_x = 0 \tag{2}$$

where $()_x$ denotes partial derivatives with respect to x and

$$\frac{\mathrm{d}^2 w}{\mathrm{d}t^2} = w_{tt} + 2vw_{xt} + v^2 w_{xx} + aw_x \tag{3}$$

where $()_t$ denotes partial derivatives with respect to time.

The displacements at the boundaries $x = 0$, $L(t)$ are defined as

$$w(0,t) = 0, \mathrm{w[L}(t),\mathrm{t]=w_L}(t) \tag{4}$$

where $w_L(t)$ represents lateral displacements of the structure corresponding to the upper end of the cable (see Fig. 1). The continuous system described by Eqs. (2)–(4) is discretized using the following approximation of the solution:

$$w(x,t) = \bar{w}(x,t) + W_0(x,t), \quad 0 \le x \le L(t) \tag{5}$$

where

$$\bar{w}(x,t) = \sum_{n=1}^{N} \Phi_n[x; L(t)]q_n(t), \quad 0 \le x \le L(t) \tag{6}$$

is an approximate solution that satisfies homogenous boundary conditions with $\Phi_n[x; L(t)]$ representing the nth eigenfunction of a taut string of instantaneous length $L = L(t)$ with a constant tension. The eigenfunctions are given as

$$\Phi_n[x; L(t)] = \sin\frac{n\pi}{L(t)}x, \quad n = 1, 2, \ldots, N \tag{7}$$

and $q_n(t)$ represents the nth modal coordinate. $W_0(x,t)$ is a particular solution that satisfies the non-homogenous boundary conditions (4). Noting that the lateral displacements at $x = L(t)$ can be expressed as $w_L(t) = \Psi_L[L(t)]w_0(t)$, where $\Psi_L = \Psi[L(t) - l]$, the particular solution is given as

$$W_0(x,t) = \Upsilon[x; L(t)]w_0(t), \quad 0 \le x \le L(t) \tag{8}$$

where $\Upsilon[x; L(t)] = \Psi_L[L(t)]\frac{x}{L(t)}$. In this model, the deformation shape function $\Psi(z)$ is assumed to be related to the fundamental mode of the structure and is approximated by a cubic polynomial as follows:

$$\Psi(z) = 3\left(\frac{z}{z_0}\right)^2 - 2\left(\frac{z}{z_0}\right)^3 \tag{9}$$

so that the deformation shape at $z = L(t) - l$ is expressed as

$$\Psi_L[L(t)] = 3\left(\frac{L(t)-l}{z_0}\right)^2 - 2\left(\frac{L(t)-l}{z_0}\right)^3 \tag{10}$$

respectively. It can be assumed that the length L is a *slowly varying* parameter, i.e. that its variation is observed on a *slow time* scale defined as $\tau = \varepsilon t$, where $\varepsilon \ll 1$ is a small quantity [15]. Thus, $L = L(\tau)$ and the rate of change of L with respect to time t is proportional to ε

$$\frac{dL}{dt} \equiv \dot{L} = \frac{dL(\tau)}{d\tau}\frac{d\tau}{dt} = \varepsilon\frac{dL(\tau)}{d\tau}; \quad \ddot{L} = \varepsilon^2\frac{d^2L(\tau)}{d\tau^2} \tag{11}$$

Consequently, noting that $v \equiv \dot{L}$, $a = \ddot{L}$ the velocity and acceleration can also be considered as being slowly varying. Using Eq. (11) the expressions for partial derivatives with respect to time t of the expression (5) are given as

$$w_t = \sum_{n=1}^{N}\left\{\varepsilon\frac{dL(\tau)}{d\tau}\frac{\partial\Phi_n[x;L(\tau)]}{\partial L}q_n(t) + \Phi_n[x;L(\tau)]\dot{q}_n(t)\right\} + \varepsilon\frac{dL(\tau)}{d\tau}\frac{\partial\Upsilon[x;L(\tau)]}{\partial L}w_0(t) + \Upsilon[x;L(\tau)]\dot{w}_0(t); \tag{12a}$$

$$w_{tt} = \sum_{n=1}^{N}\left\{\begin{aligned}&\varepsilon^2\left[\left(\frac{dL(\tau)}{d\tau}\right)^2\frac{\partial^2\Phi_n[x;L(\tau)]}{\partial L^2} + \frac{d^2L(\tau)}{d\tau^2}\frac{\partial\Phi_n[x;L(\tau)]}{\partial L}\right]q_n(t)\\ &+2\varepsilon\frac{dL(\tau)}{d\tau}\frac{\partial\Phi_n[x;L(\tau)]}{\partial L}\dot{q}_n + \Phi_n[x;L(\tau)]\ddot{q}_n\end{aligned}\right\} + \varepsilon^2\left\{\left(\frac{dL(\tau)}{d\tau}\right)^2\frac{\partial^2\Upsilon[x;L(\tau)]}{\partial L^2} + \frac{d^2L(\tau)}{d\tau^2}\frac{\partial\Upsilon[x;L(\tau)]}{\partial L}\right\}w_0(t) + 2\varepsilon\frac{dL(\tau)}{d\tau}\frac{\partial\Upsilon[x;L(\tau)]}{\partial L}\dot{w}_0(t) + \Upsilon[x;L(\tau)]\ddot{w}_0(t); \tag{12b}$$

$$\bar{w}_{xt} = \sum_{n=1}^{N}\left\{\varepsilon\frac{dL(\tau)}{d\tau}\frac{\partial\Phi'_n[x;L(\tau)]}{\partial L}q_n(t) + \Phi'_n[x;L(\tau)]\dot{q}_n(t)\right\} + \varepsilon\frac{dL(\tau)}{d\tau}\frac{\partial\Upsilon'[x;L(\tau)]}{\partial L}w_0(t) + \Upsilon'[x;L(\tau)]\dot{w}_0(t), \tag{12c}$$

where the primes denote partial derivatives with respect to x. By using (6)–(12) in (5), substituting the result in (2), multiplying by $\Phi_r[x; L(\tau)]$,