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Analytical closed-form expressions for the structural response of helical constructions to thermal loads

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ABSTRACT

In the current work, we present an analytical model for the characterization of the mechanical response of helical constructions to thermal loads. In particular, we elaborate closed-form expressions to compute the forces and moments induced by homogeneous and non-homogeneous temperature fields. We numerically verify the validity of the formulas over a wide range of geometric configurations, while we apply them to evaluate the mechanical response of two single layer cable configurations upon combined axial and thermal strains. Finally, we demarcate the range of angles for which the thermal structural response to homogeneous and non-homogeneous temperature fields considerably differs.

1. Introduction

Helices are encountered in biological constructions, as for example in tendons [1], as well as in a wide range of engineering applications. In particular, they are used as the primal constituent of cables, ropes and stents [2–6], while they are commonly found as reinforcement, or nano- and meso-structure elements [7–9].

In order to understand and characterize their mechanical behavior, a large number of analytical models have been developed. The models have provided closed-form expressions that relate the helix material and geometric attributes with the forces and moments induced by different loading patterns. More specifically, formulas characterizing the mechanical response to axisymmetric loads, in particular to axial [10], torsional [11–13] and radial loads [14] have been elaborated. Moreover, closed-form expressions for the analysis of non-axisymmetric loads as for example bending loads have been derived [15].

The analytical models have been complemented by both laboratory experiments and numerical models. On the experimental modeling side, the mechanical response of single and multi-layer helical strands to axial loads has been measured [16,17]. The measurements have provided useful insights into the role of the strand's structural arrangement on the deformation profile, as well as on the forces and moments generated. On the numerical modeling side, different finite element models have been developed to simulate the helix axial and torsional loading mechanical response [18–20], their applicability being primarily illustrated in the context of engineering helical strands [21].

In the realm of biomechanical engineering, clinical studies have been conducted to characterize the response of metallic helical stents to pressure radial loads [22]. Moreover, dedicated finite element models of helical scaffolds have been constructed to provide a virtual bioengineering framework for the design of biocompatible ligament tissue engineering scaffolds [23,24].

Dedicated numerical models have been further constructed to analyze the axial loading mechanical properties of nanoscale helical springs with different cross section profiles [25], noting their extensive applications in nanoengineering [26], as well as in microsystem technologies [27].

Along with the need to analyze and characterize the response of helical structures to mechanical loads (e.g. axial, torsional or radial loads), the necessity to account for the effect of non-mechanical loads, in particular of thermal effects has become apparent in a wide range of applications. More specifically, in power transmission cable engineering, variations in the electrical load induce thermal loads, which in turn alter the structure's loading state and considerably affect its life expectancy [28,29]. Accordingly, in composite structure's engineering, in order to appropriately design helical bar reinforcements, the range of allowable temperature variations needs to be determined, for which analytical effective rod models and numerical models have been

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proposed [7]. Effective layered cylinder models have been respectively developed to retrieve the thermal mechanical properties of carbon nanotube arrays [8]. Finally, the effect of thermal loads on the structural response of suspended cables has been elaborated, making use of the catenary equation [6].

While a large number of analytical and numerical models have been developed to characterize the structural response of helical constructions to mechanical loads, the effect of non-mechanical loads, in particular of thermal loads, has been captured by means of either effective analytical models, or numerical models. However, no analytical expression has been so far provided to explicitly account for the role of the helix geometric and material properties, as well as for the thermal loading input type.

In the current work, we derive closed-form expressions that characterize the structural response of helical constructions to homogeneous and non-homogeneous thermal loads. In Section 2, we describe the helix thermal loading kinematics, upon which, we formulate the structure's constitutive and equilibrium equations. By that means, we correlate the helix material and geometric attributes with the forces and moments developed, providing in Section 3.1 analytical thermal loading stiffness expressions. In Section 3.2, we employ the formulas to assess the effect of different thermal loads on the loading state of single layer, axially strained cables. Finally, we discuss on the magnitude of the forces and moments created in conjunction to the thermal field applied and conclude with a discussion of the main results in Section 4.

2. Mechanical model formulation

2.1. Helix geometry

We describe the helix geometry as a helical fiber. The helix centerline is characterized by the position vector $\mathbf{R}(\varphi)$, defined in a *Cartesian* basis $\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$ as follows [30]:

$$\mathbf{R}(\varphi) = \begin{Bmatrix} a\cos\varphi \\ a\sin\varphi \\ b\,\varphi \end{Bmatrix}, \quad \varphi = \frac{\ell}{\gamma}, \quad \gamma = \sqrt{a^2+b^2} \quad b = a\tan\theta \tag{1}$$

In Eq. (1), parameter a represents the helix radius, while b stands for the helix rise along the central *Cartesian* axis z and γ for the helix *curvilinear* length per unit angular evolution φ. We schematically depict the afore-introduced helix geometric parameters in Fig. 1:

The parameter b is related to the helix radius a through the tangent of the angle θ (Eq. (1)), while the helix cross section radius is denoted with r (Fig. 1). The Serret–Frenet *curvilinear* $\mathbf{n}$, $\mathbf{b}$, $\mathbf{t}$ basis is thereupon given as [30]:

$$\mathbf{n} = \begin{bmatrix} -\cos\varphi \\ -\sin\varphi \\ 0 \end{bmatrix}, \quad \mathbf{b} = \frac{1}{\gamma}\begin{bmatrix} b\sin\varphi \\ -b\cos\varphi \\ a \end{bmatrix}, \quad \mathbf{t} = \frac{1}{\gamma}\begin{bmatrix} -a\sin\varphi \\ a\cos\varphi \\ b \end{bmatrix} \tag{2}$$

The helix curvature and tortuosity named respectively as k and τ are defined by means of the helix radius a and helix angle θ as follows [30]:

$$k = \frac{\cos^2\theta}{a}, \quad \tau = \frac{\sin\theta\cos\theta}{a} \tag{3}$$

Finally, the *curvilinear* helix length ℓ of a non-unit angular evolution φ and its projection on the *Cartesian* axis z, named h, are related to the helix radius a and helix angle θ by the following geometric relations [11]:

$$h = \ell\sin\theta, \quad a\varphi = \ell\cos\theta \tag{4}$$

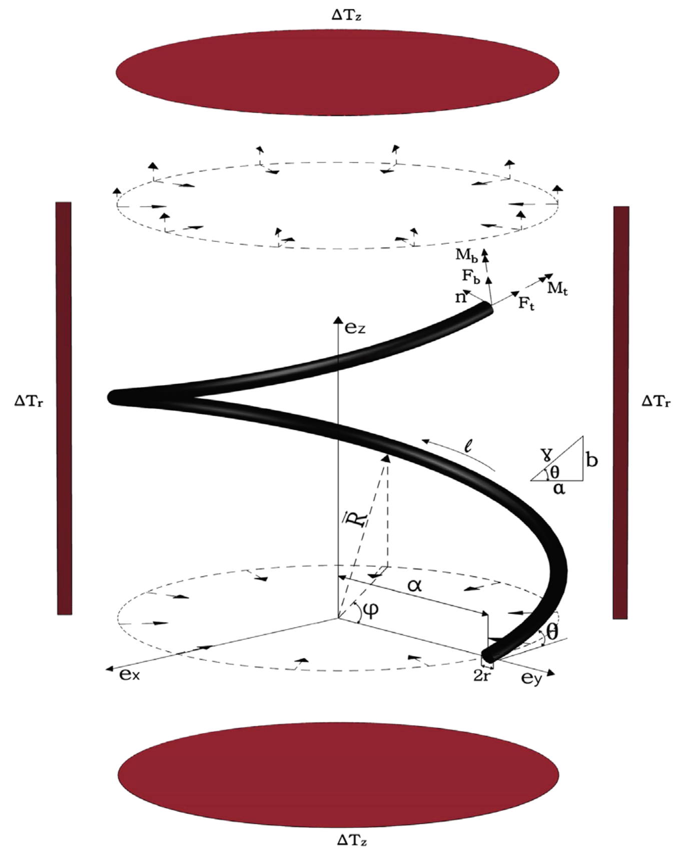

Fig. 1. Helix geometry.

2.2. Helix structural response definition

We describe the helix structural response by means of three Quantities of Interest (*QoI*), namely the axial force F_z and moment M_z created along the *Cartesian* axis z and the radial force F_r. To compute the effective structural response $\left[F_z\ M_z\ F_r\right]^T$, we distinguish between the mechanical loading contributions (e.g. axial, torsional or radial strains) $\left[F_z\ M_z\ F_r\right]^T_m$ and the non-mechanical loading contributions $\left[F_z\ M_z\ F_r\right]^T_{th}$ induced by thermal strains:

$$\begin{bmatrix} F_z \\ M_z \\ F_r \end{bmatrix} = \begin{bmatrix} F_z \\ M_z \\ F_r \end{bmatrix}_m - \begin{bmatrix} F_z \\ M_z \\ F_r \end{bmatrix}_{th} \tag{5}$$

In the linear superposition of Eq. (5), the thermal part contribution (th) is subtracted from the mechanical part (m), as positive signed thermal and positive signed mechanical strains induce opposite orientation force and moment resultants [31].

The structural response to mechanical strains is given in Appendix B. We hereby derive the helix thermal loading closed-form stiffness expressions, relating the thermally induced force and moment resultants of Eq. (5) to the applied thermal strains. To that extent, we express the thermal part $\left[F_z\ M_z\ F_r\right]^T_{th}$ of Eq. (5) as the multiple of a thermal loading stiffness matrix $\boldsymbol{\kappa}_{th}$ with the thermal strain vector $\boldsymbol{\epsilon}^{th} = \left[\epsilon^{th}_x,\ \epsilon^{th}_y,\ \epsilon^{th}_z\right] = \left[\alpha_{th}\Delta T_x,\ \alpha_{th}\Delta T_y,\ \alpha_{th}\Delta T_z\right]$, where α_{th} stands for the linear thermal expansion coefficient (considering an isotropic thermal behavior with equal expansion coefficients) and ΔT_x, ΔT_y, ΔT_z for the temperature changes along the *Cartesian* x , y and z axes. The thermal *QoI* in Eq. (5) are thereupon expressed as: