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Asymptotic analysis of asymmetric thin sheet rolling

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ABSTRACT

An analytical model for asymmetric rolling is presented, which includes asymmetry in roll friction, roll size and roll speed, for a rigid, perfectly plastic thin sheet deformed with Coulomb friction. This model is solved asymptotically, based on the systematic assumptions that both the roll gap aspect ratio and the friction coefficient are small. While the leading order solution is shown to be consistent with an existing slab model, we are able to derive additional detail by looking to higher orders. We compare our higher order solution and the leading order solution with finite element simulations, and use the results to determine the practical range of validity of our analytical model. Within this region, it gives reasonable quantitative predictions of the force and torque results from finite element simulations and approximates through thickness variation of stress and strain with orders of magnitude shorter computation times. A *MATLAB* implementation of this solution is included in the supplementary material.

1. Introduction

Rolling is the process of reducing the thickness of a metal workpiece by passing it between two rotating rolls with separation less than the current workpiece thickness. This process is used to produce sheet metal and other products.

Asymmetry in rolling can arise through inequality in roll radii, roll velocity and interface friction; inhomogeneous or anisotropic workpiece material; or bending end forces. Regardless of the mechanism, asymmetry generally results in curvature of the rolled workpiece. If this is unintentional, perhaps a consequence of tool wear, it can cause compatibility difficulties and machine damage. Intentional asymmetry can be used to reduce the total roll torque and force required to achieve a given reduction and can provide flexibility in machine design.

The mechanical simplicity of driving only a single roll first motivated investigations into asymmetric rolling but it continues to be an active area of research for other reasons. In addition to process efficiency gains, improved workpiece quality and reduced maintenance requirements; curvature can also be desirable if controlled to produce a wider range of products.

Given these attractions, improving online control of asymmetric rolling is an area of interest. For use in control, solution times should not slow the processes when included in the control loop. This prohibits the use of finite element simulations and motivates research into faster, yet still accurate, analytical models.

Early experimentation has been used to quantify roll force and torque for a range of geometries and materials [3,22]. A variety of techniques have also been used to investigate other properties such as contact stress distributions and workpiece curvature [17,1,10].

Analytical work has included a range of approaches; the most popular being modifying one-dimensional 'slab' models of symmetric rolling. These models are constructed without systematic consideration of the physical system so the subsequent assumptions remain questionable. This seems particularly relevant for recent works [21,26,25,11,9] which extend models such as Hwang and Tzou [13] to capture greater asymmetry and predict curvature.

Alternative techniques have also included upper-bound methods [18,12] and slip-line methods [7,5]. While both are able to predict characteristics such as curvature, the roll contact points or the yield region, the solution processes require a priori knowledge or assumptions about the form of the solution. This hinders the development of these models for other geometries and materials, therefore they have seen less attention in recent work.

Finite element simulations have also been applied to provide detailed results for more general configurations. Most studies focus on predictions of roll force, roll torque, and workpiece curvature [27,24,20] with some more recent studies considering microstructure [23]. While impractical for online control, this approach can be used to gain understanding of the processes and facilitate inexpensive exploration of configurations. The results of these publications have also been used to validate some of the previous analytical models. Understanding the operating limits of

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a model could also be ascertained by running simulations tailored to the model in question.

Another technique has also been used in modelling symmetric rolling: asymptotic analysis. Asymptotic analysis exploits systematic assumptions of scale to find a rigorous yet tractable approximation, as opposed to simplifications through ad hoc assumptions of unknown error and limitation. A series of publications [28,16,8,4,2] develop models of rolling underpinned by an assumption of a small roll gap aspect ratio: the sheet is much thinner than the length of the roll gap. Without breaking this assumption a range of geometries, friction models and material models have been implemented.

This technique has only been applied to asymmetric rolling by Johnson [15] where asymmetries were only considered for the friction coefficients and roll speeds. The friction coefficient was assumed to be an order of magnitude larger than the roll gap aspect ratio which is not representative of many thin sheet processes which are predominantly cold rolling. Experiments [17] and simulations [19,20] also show the sign of curvature can be dependent on geometry indicating that specific account of roll size may be necessary to capture the complete dynamics of the process.

In the interest of developing analytical models with sufficient resolution to potentially make curvature and microstructure predictions, the present work develops an asymptotic model to explicitly include asymmetric roll size with asymmetric roll speed and asymmetric, small magnitude friction. Complete stress and strain fields are achieved with this approach. In Section 2 we present a model of asymmetric rolling assuming a rigid, perfectly plastic workpiece and roll-workpiece interaction driven by slipping Coulomb friction. The choice of material and friction models is for illustration only; by analogy, a solution could be found for any of the friction or material models in the literature [28,16,8,4,2]. This model is non-dimensionalised to find six non-dimensional groups: the aspect ratio, δ, and the friction coefficient μ, which are assumed to be small; the sheet reduction r; and the ratios of roll size, speed and friction, which are considered unrestricted. An asymptotic solution to this model comprises Section 3, and the model is validated against the explicit solver of the commercial finite element package *ABAQUS* through a range of asymmetries and parameters in Section 5.

2. Model formulation

We assume a plane strain configuration, which is valid away from the workpiece edges for sufficiently wide workpieces. Hence, Fig. 1 captures the extent of the model.

The rolls are vertically aligned and the workpiece is fed horizontally. The initial workpiece half thickness is $\hat{h}_0$ and the length of the roll gap is $\hat{l}$, giving the aspect ratio as $\delta = \hat{h}_0/\hat{l}$ where δ is assumed to be small. This is most appropriate when considering a thin sheet with large or flattened working rolls.

The material model is assumed rigid perfectly plastic; that is, no elasticity and no hardening. We assume that plastic deformation occurs everywhere in the roll gap and adopts vertical boundaries at the entry and exit (marked as the hashed region in Fig. 1). This assumption is typical of existing 'slab' and asymptotic models of rolling.

Although assuming vertical boundaries to the plastic region imposes very specific combinations of bending and shear end conditions for a given asymmetry, it has been shown experimentally that the bending effects from non-extreme end conditions can be neglected [26].

The von Mises yield criteria and associated flow rule, the Levy–Mises equations, are used and slipping Coulomb friction describes the roll-workpiece interaction like Domanti and McElwain [8]. Unlike Domanti and McElwain [8], rather like the first model of Cherukuri et al. [4], the friction coefficient is also assumed to be small, $\mu \ll 1$. Asymmetry is introduced into the friction coefficient, μ; roll radius, $\hat{R}$; and rotational speeds, $\hat{\Omega}$, which must all be defined for the top, subscripted t, and bottom, subscripted b, rolls separately. These assumptions are valid for foil rolling, but may also be valid in other regimes that fall within these assumptions.

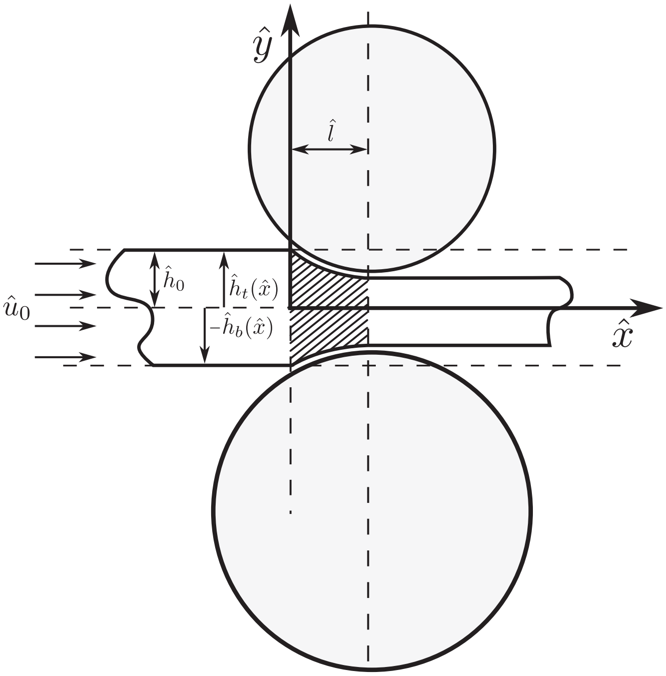

Fig. 1. Illustration of the idealised two dimensional rolling model.

The velocity on the roll surfaces is restricted by the no-penetration condition. Horizontal and vertical force equilibria on the roll surfaces are combined with the Coulomb friction model to give the shear boundary condition on the top and bottom roll. The model is closed by applying a given force at each end of the roll gap.

Using carets to denote dimensional quantities, we can define $\hat{p}$, $\hat{s}_{ij}$, $\hat{u}$, $\hat{v}$, $\hat{\lambda}$ and $\hat{k}$ as the pressure, ijth deviatoric stress, horizontal velocity, vertical velocity, flow parameter and yield stress respectively. Also, $\hat{h}_{t/b}(\hat{x})$ is the roll surface, applicable to both top and bottom rolls and $\hat{F}_{\text{in/out}}$ are the end tensions, per unit width, applied to the workpiece, applicable to the upstream and downstream workpiece.

We define the upstream velocity of the workpiece as $\hat{u}_0$, although it is not possible to specify it independently of the two roll velocities. Consequently, we shall consider $\hat{u}_0$ to be the characteristic velocity for the purpose of non-dimensionalisation then determine its value from the roll velocities.

2.1. Non-dimensionalisation

We scale vertical distances with the initial workpiece half thickness, $\hat{h}_0$, and horizontal distances with the length of the roll gap, $\hat{l}$. The aspect ratio, $\delta = \hat{h}_0/\hat{l}$, is assumed to be small. As the friction is also small, $\mu_{t/b} = O(\delta)$, we define a normalised friction coefficient, $\beta = \mu_b/\delta = O(1)$ such that δ is the sole small parameter.

Using the scaling choice of Cherukuri et al. [4], the shear stress scales with the friction coefficient and yield stress, $\hat{s}_{xy} = \delta\beta\hat{k}s_{xy}$.