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Free edge stresses in composite laminates with imperfect interfaces under extension, bending and twisting loading

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ABSTRACT

An accurate analytical three-dimensional elasticity solution is presented for the free edge stresses in composite laminates under extension, bending and twisting loading, accounting for possible bonding imperfections at the layer interfaces. The bonding imperfection is modeled using the linear spring-layer model. A mixed formulation with both displacements and stresses as primary variables is developed using the Reissner-type variational principle and the solution is obtained using the recently developed mixed-field multiterm extended Kantorovich method (MMEKM). The method ensures exact and point-wise satisfaction of the traction-free edge boundary conditions and interlaminar stress continuity conditions, which is key to the accurate solution of free edge stresses. A detailed numerical study established excellent convergence characteristics of iterative series solution. The presence of bonding imperfection at an interface leads to nonsingular free edge interlaminar stresses at that interface, in contrast with the singular stress field occurring at a classical perfect interface. The effect of loading type on the relative reduction of free edge stresses with the increase in imperfection compliance is investigated for cross-ply and angle-ply laminates. The effect of ply-angle of angle-ply laminates on the same is also ascertained.

1. Introduction

The free edge effect in laminated composite structures has been a topic of intense research for almost five decades now [1,2]. Its importance in the design of laminated structures arises from the fact that the development of large localized out-of-plane stresses at the layer interfaces in the vicinity of free edges can lead to delamination, a common mode of failure in such structures. These localized stresses caused by the material and geometric discontinuities that exist at the free edge interfaces are truly three-dimensional (3D) in nature and decay rapidly away from the edges. Although there have been numerous studies on the determination of the 3D free edge stress field in composite laminated panels and plates, almost all of them assumed the layer interfaces to be perfectly bonded, and there has been no study ascertaining the effect of bonding imperfections on the free edge stresses in composite structures under different loadings. This work is aimed at developing an accurate 3D elasticity solution of the free edge stress field in imperfectly bonded laminated panels subjected to extension, bending and twisting loading.

The search for accurate 3D elasticity based solutions for the free edge problem started with the work of Pipes and Pagano [3] in 1970, who employed the finite difference method to obtain the free edge stress field in perfectly bonded angle-ply laminates under uniaxial extension. Following this work, various numerical methods such as the finite element method (FEM) [4–8], and the boundary element method [9] have been employed for this problem. Since the free edge stresses often have singularity and show sharp variation near the edges, the accuracy of the results is not ensured by numerical methods. Hence, several approximate analytical solutions have been presented. Cho and Yoon [10] presented an iterative analytical solution for free edge stresses in composite laminates under extension loading using the Lekhnitskii stress function and the extended Kantorovich method (EKM) [11]. This method has been subsequently extended for bending, twisting and thermal loading [12], strength analysis of laminates [13], and free edge stresses in bonded composite patches [14]. A displacement based formulation with the EKM solution has been presented by Andakhshideh and Tahani [15,16] for the free edge stress analysis of finite rectangular plates under extension, shear, bending, twisting and thermal loading. In the stress based formulations, the displacement continuity conditions at the interfaces can be satisfied only in an integral sense, whereas in displacement based solutions, the point-wise satisfaction of the traction-free edge conditions and interlaminar stress continuity conditions are not feasible, which are essential for ensuring the accuracy of the predicted stresses. In order to overcome these shortcomings of the

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stress based and displacement based formulations, a mixed-field 3D elasticity solution based on the multiterm EKM has been presented by the authors for free edge stresses in laminated composite panels under extension, bending, twisting and thermal loading [17]. The mixed-field formulation uses both displacements and stresses as primary variables and thereby ensures the satisfaction of all boundary and interface boundary conditions exactly and point-wise. It also ensures the same order of accuracy for the predicted displacements and stresses.

All the aforementioned studies on the free edge stress field in composite laminates assume perfect bonding (rigid bonding) between the adjacent plies. However, in many practical cases, damages such as flaws, microcracks, voids and imperfect adhesion may get introduced at the layer interfaces during manufacturing process itself or during service conditions (fatigue loading, chemical reactions, environmental attack) [18,19]. The assumptions of interlaminar displacement and stress continuity for the perfectly bonded interfaces are no longer valid at these weak interfaces. Such bonding imperfections in laminates have been generally modeled [20–22] by relating the displacement discontinuities at the weak interface to the corresponding interlaminar stress components. A very popular model is the simple linear spring-layer model, which assumes a linear relationship between the displacement jumps and the interlaminar stresses [23–26]. This model has been used for obtaining exact 3D elasticity solutions for bending and free vibration of simply-supported cross-ply composite rectangular plates and angle-ply panels [27,28], buckling response of simply supported cross-ply rectangular plates [29] and thermo-electro-mechanical response of orthotropic piezoelectric laminated rectangular plates [30], featuring imperfectly bonded interfaces. Such exact 3D elasticity analytical solutions are not possible for weakly bonded laminated structures with non-simply supported boundary conditions, which are essential to understand the so-called edge effect or the boundary layer effect. Recently, Kapuria and Dhanesh [31] employed the mixed-field multiterm extended Kantorovich method (MMEKM) to obtain an accurate 3D elasticity solution for the cylindrical bending of weakly bonded composite and sandwich panels having arbitrary boundary conditions under transverse loading, using the spring-layer model for weak bonding.

Such accurate analytical solutions for the free edge stress field in imperfectly bonded composite laminates under various loadings are not known to exist, to the best of authors' knowledge. Haboussi et al. [32,33] proposed two models for accounting for the compliance of the interfacial zone between adjacent layers in composite laminates and employed them in the free edge stress analysis of cross-ply laminates in extension using the commercial FE software ABAQUS. The first model, called the transition behavior law, treats the interface as a separate interface layer of graded stiffness properties between the two adjacent plies, while the second model, utilizes an asymptotic interface law, akin to the spring-layer model. Kim et al. [34] modeled the thin adhesive/interface layer between the laminas to predict the free edge stress field in laminates under thermomechanical loading using their stress function based EKM solution [10,12]. These results showed that the singularity observed in the free edge stress field in the perfect bonding case is eliminated when the interfacial compliance is considered. In another development, the thin adhesive layers have been modeled as imperfect interfaces with elasto-plastic properties in the analysis of double lap and T-peel joints using layerwise stress based analytical [35] and finite element models [36].

In this work, we present an accurate analytical 3D elasticity solution for free edge stresses in imperfectly bonded laminated composite panels under extension, bending and twisting loadings. The recently developed 3D elasticity based MMEKM solution for free edge stresses in perfectly bonded laminates [17] is extended to incorporate imperfect interfacial conditions using the linear spring-layer model. The governing equations for the mixed-field formulations are developed using the Reissner-type mixed variational principle for imperfectly bonded laminates [31]. The formulation incorporates different loading conditions in the form of applied strain and curvatures, using Lekhnitskii's displacement and strain fields [37]. The convergence of the iterative MMEKM solution is studied for the different loadings. For validation, the present results are compared with detailed finite element (FE) solutions, as published results on the same are not available in the literature. Further, the effects of loading, ply-angle and location of the imperfect interface on the variation of free edge stress field are investigated.

2. Theoretical formulation

2.1. Governing equations for laminas

A multilayered panel (Fig. 1) made of L number of angle-ply fiber reinforced composite layers with parallel free edges at $x=0$

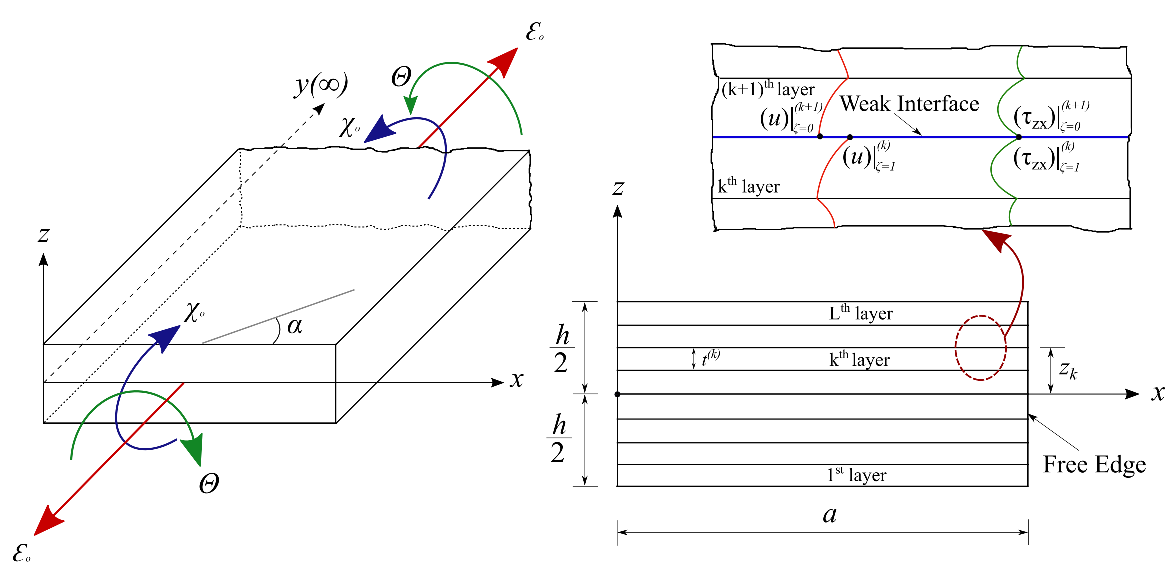

Fig. 1. Geometry of the laminated panel featuring interfacial imperfection with free edges subjected to extension, bending and twisting loading.