International Journal of Mechanical Sciences 110 (2016) 22–33

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Post-wrinkling analysis of a torsionally sheared annular thin film by using a compound series method

C.G. Wang [a,b,*], Y.P. Liu [a,b], L. Lan [c], L. Li [d], H.F. Tan [a,b]

[a] Center for Composite Materials, Harbin Institute of Technology, Harbin 150080, China
[b] National Key Laboratory of Science and Technology on Advanced Composites in Special Environments, Harbin Institute of Technology, Harbin 150080, China
[c] Shanghai YS Information Technology Co., Ltd., Shanghai 200240, China
[d] Department of Materials Science and Engineering (MSE), Cornell University, Ithaca, NY 14850, USA

ARTICLE INFO

Article history:
Received 11 December 2015
Received in revised form
18 February 2016
Accepted 19 February 2016
Available online 2 March 2016

Keywords:
Thin film
Buckling
Wrinkle
Compound series
Finite difference

ABSTRACT

This paper investigates the torsional wrinkling behavior of an annular thin film. Non-dimensional nonlinear von Karman buckling equations are established, which are solved by introducing a compound series method to acquire the post-wrinkling characteristics. The proposed theoretical model can accurately predict the critical wrinkling behavior and post-wrinkling characteristics of the annular thin film, which are verified by the experimental measurement based on the digital image correlation (DIC) technique. The theoretical results show that the post-wrinkling stress is intimately associated with the wrinkle configuration in the post-wrinkling stage. The hoop post-wrinkling stress along the wrinkle texture direction of the annular thin film dictates the wrinkle evolution. The wrinkle number remains constant in the elastic regime, which is determined by the critical buckling load factor. The results provide good guides to tune or control the wrinkles in the thin film.

1. Introduction

Wrinkling is a common phenomenon that may occur in a thin film as long as a slight compressive stress is present. The first discussion on the wrinkling phenomenon could date back to 1929, when Wagner carried out a series of wrinkling calculations on a thin metal sheet under shearing [1]. Recently, thin film wrinkling has provoked extensive research owing to its importance in a wide variety of applications, ranging from deployable gossamer spacecraft [2,3], biological tissues [4–6], to nanoscale devices [7–11]. Note that wrinkling, on one hand, is a nuisance in some cases. For example, in the aerospace industry researchers raise serious concerns about the adverse effects of wrinkles on the performance and longevity of a thin structural element [2,12], because the onset of wrinkles may significantly influence its static configuration and dynamic behavior [3,13–16]. On the other hand, wrinkling may also be used as an efficient way of tuning the properties of a nanoscale material [7,17–21], particularly its mechanical properties [22,23]. In this context, it is of great importance to understand the mechanics of thin film wrinkling in areas where reliable control and tuning of wrinkles are required.

In principle, wrinkling refers to the local buckling instability of a thin film that has a small but finite bending stiffness. Thin film wrinkling is generally considered as a bifurcation from plane stress deflection to out-of-plane bending-type deflection. It is found that wrinkles are in constant development under increasing external load in the post-wrinkling stage [15,24–27]. As such, an efficient analytical model capable of dealing with large deflection, based on the nonlinear post-buckling theory, is in vital demand, aiming to exactly capture the wrinkle characteristics of a thin film in the post-wrinkling stage.

Thus far, a number of numerical studies, mainly based on nonlinear shell buckling simulations, have been conducted for the wrinkling calculations [12,15,24–26,28–30]. In these numerical studies, the thin shell elements are exclusively assumed to have bending and membrane stiffness that facilitates efficient deduction of detailed wrinkle characteristics such as wrinkle number, amplitude and wavelength. It is worth mentioning, however, that a singular tangent stiffness matrix is regrettably encountered near the bifurcation point, which may result in poor convergence of the wrinkling calculations. Moreover, the wrinkling calculations based on nonlinear shell buckling simulations are highly mesh-dependent and sensitive to the initial imperfection, which are not beneficial for the accurate prediction of the wrinkle characteristics in the post-wrinkling stage [15,25,28,31,32]. Therefore, theoretical studies on the post-wrinkling behavior of a thin film

* Corresponding author at: Center for Composite Materials, Harbin Institute of Technology, Harbin 150080, China.
E-mail address: wangcg@hit.edu.cn (C.G. Wang).

http://dx.doi.org/10.1016/j.ijmecsci.2016.02.011
0020-7403/© 2016 Elsevier Ltd. All rights reserved.

are in urgent need, enabling accurate prediction of the wrinkle characteristics in the post-wrinkling stage.

Unfortunately, there are very few theoretical studies on the post-wrinkling behavior of a thin film at present [33–36], in contrast to the numerous linear buckling theoretical studies [37–43]. It should be stressed, however, that although the linear buckling theory can efficiently identify the eigenmode and critical wrinkling load of a thin film, it fails to determine the wrinkle amplitude where the nonlinear effect should be taken into account. Coman and coworkers [38–42] found that the wrinkle wavelength remained the same as there was no "branching" in the initial post-wrinkling stage, which was in good agreement with what was concluded by Geminard *et al.* [37]. As suggested by Wang *et al.*, [25,28,44] thin film wrinkling was regarded as out-of-plane bending-type deflection induced by the post-wrinkling stress, and was a kind of typical nonlinear large deflection that could be found in various special applications [3,8,15,25,35,45]. In consequence, the nonlinear large deflection of a wrinkled thin film, i.e., thin film wrinkling, should be described by the nonlinear theory proposed by von Karman. The approximate solutions to von Karman's equations can be obtained by using perturbation methods [38,39], asymptotic expansions [40–42], finite difference methods [33–36], or asymptotic numerical method [46,47].

Thus far, little attention has been paid to the theoretical studies on the post-wrinkling behavior of a thin film, mainly owing to the difficulties in the mathematical modeling and solution towards this mechanical problem. Besides, experimental evidence has proven that the wrinkle characteristics predicted by the linearized Donnell-von Karman bifurcation theory are far from being satisfactory [26,28,32]. Given the above review, hereby we aim to develop a nonlinear theoretical analysis for the post-wrinkling of a stretched annular thin film by extending the present work [40,41] to the post-wrinkling stage. The stress state is not simply a hoop periodic function in the post-wrinkling stage, but a function coupled with the wrinkling configuration. Nonlinearity of modeling and solution in the post-wrinkling stage become more prominent. In order to have a more accurate prediction of post-wrinkling characteristics, this paper is structured as follows: In Section 2, non-dimensional nonlinear von Karman buckling equations are established, which are solved by introducing a compound series method to acquire the post-wrinkling characteristics of a thin film, including post-wrinkling stress and wrinkle configuration intimately associated with it. In Section 3, experiments are conducted on the torsional wrinkling of an annular thin film to verify the prediction accuracy of the proposed theoretical model. In Section 4, the critical wrinkling load, post-wrinkling stress distribution and variation, and detailed wrinkle characteristics are predicted based on the theoretical model. The conclusions are drawn in the end.

2. Mathematical description of the mechanical problem

At first, we consider a model of an annular thin film under inner torsion and outer tension, as illustrated in Fig. 1. As seen from the figure, the inner and outer radii of the annular thin film are denoted as r_1 and r_2, respectively. The outer boundary of the annular thin film is stretched initially, resulting in a uniform displacement field of u_0. Meanwhile, a torque M is exerted on the inner boundary of the annular thin film. A cylindrical system of coordinates (r, ϕ, z) is applied to define all variables associated with this mechanical problem.

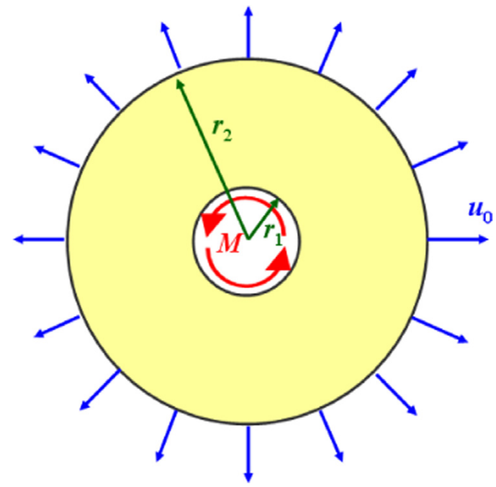

Fig. 1. A model of an annular thin film under inner torsion and outer tension.

2.1. Nonlinear wrinkling of an annular thin film

Thin film wrinkling, in its nature, is a nonlinear, large out-of-plane deflection that is intimately associated with a post-bifurcation stress field. Therefore, the physical framework of thin film wrinkling, different from that addressed by the linear bifurcation analysis where the stress field of a base state of interest is considered, should be clarified by the nonlinear theory proposed by von Karman.

Nonlinear von Karman buckling equations taking into account of large deflection can be described as follows:

$$\nabla^4 w = \frac{h}{D}\left[\sigma_{rr}\frac{\partial^2 w}{\partial r^2} + 2\sigma_{r\varphi}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial w}{\partial \varphi}\right) + \sigma_{\varphi\varphi}\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \varphi^2}\right)\right] \tag{1a}$$

$$\nabla^4 \Phi = E\left[\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial w}{\partial \varphi}\right)\right)^2 - \frac{\partial^2 w}{\partial r^2}\cdot\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \varphi^2}\right)\right] \tag{1b}$$

where E and v are Young's modulus and Poisson's ratio, respectively. h is the thickness of the annular thin film. $D = Eh^3/12(1-v^2)$ is the bending stiffness of the annular thin film. ∇^2 represents the Laplace operator in the polar coordinates. $w(r, \varphi)$ is the out-of-plane displacement, and $\Phi(r, \varphi)$ is the stress function.

Several non-dimensional parameters, as defined by the following formula, are substituted in Eqs. (1a) and (1b):

$\rho = r/r_2, \eta = r_1/r_2, \overline{w}(\rho, \varphi) = w(r, \varphi)/r_2, \chi = 12r_2^2(1-v^2)/h^2$,-$\overline{\Phi}(\rho, \phi) = \Phi(r, \phi)/r_2^2 E$

We can obtain the following two equations:

$$\nabla^4 \overline{w} = \chi\left[\overline{\sigma}_{\rho\rho}\frac{\partial^2 \overline{w}}{\partial \rho^2} + 2\overline{\sigma}_{\rho\varphi}\frac{\partial}{\partial \rho}\left(\frac{1}{\rho}\frac{\partial \overline{w}}{\partial \varphi}\right) + \overline{\sigma}_{\varphi\varphi}\left(\frac{1}{\rho}\frac{\partial \overline{w}}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 \overline{w}}{\partial \varphi^2}\right)\right] \tag{2a}$$

$$\nabla^4 \overline{\Phi} = \left[\left(\frac{\partial}{\partial \rho}\left(\frac{1}{\rho}\frac{\partial \overline{w}}{\partial \varphi}\right)\right)^2 - \frac{\partial^2 \overline{w}}{\partial \rho^2}\cdot\left(\frac{1}{\rho}\frac{\partial \overline{w}}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 \overline{w}}{\partial \varphi^2}\right)\right] \tag{2b}$$

where

$$\begin{cases} \overline{\sigma}_{\rho\rho}(\rho, \phi) = \frac{1}{\rho}\frac{\partial \overline{\Phi}(\rho, \phi)}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 \overline{\Phi}(\rho, \phi)}{\partial \phi^2} \\ \overline{\sigma}_{\phi\phi}(\rho, \phi) = \frac{\partial^2 \overline{\Phi}(\rho, \phi)}{\partial \rho^2} \\ \overline{\sigma}_{\rho\phi}(\rho, \phi) = -\frac{\partial}{\partial \rho}\left(\frac{1}{\rho}\frac{\partial \overline{\Phi}(\rho, \phi)}{\partial \phi}\right) \end{cases} \tag{3}$$

The boundary conditions are as follows:

$\overline{w}(\rho, \varphi)|_{\rho=\eta} = \overline{w}(\rho, \varphi)|_{\rho=1} = 0,\ \partial \overline{w}(\rho, \varphi)/\partial \rho|_{\rho=\eta}$