International Journal of Mechanical Sciences 110 (2016) 34–40

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Macro/micro simultaneous validation for multiscale analysis of semi-periodically perforated plate using full-field strain measurement

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ARTICLE INFO

Article history:
Received 18 August 2015
Received in revised form
31 December 2015
Accepted 13 February 2016
Available online 18 February 2016

Keywords:
Multiscale analysis
V&V
Digital image correlation
Homogenization
Periodicity

ABSTRACT

Multiscale analysis of a semi-periodically perforated plate based on a homogenization theory is experimentally validated both macroscopically and microscopically, using a full-field strain measurement. To do this, a plate-fin-type perforated plate with a misaligned microstructure is considered as a semi-periodically perforated plate. Then, a homogenization theory that can analyze macroscopic behavior and microscopic stress and strain distributions of the perforated plate is presented. To validate the theory, a tensile test of a plate-fin-type semi-periodically perforated plate made of epoxy resin is conducted. During the test, the microscopic deformation of the specimen is observed with a digital image correlation (DIC) full-field measurement system, from which microscopic strain distribution of the specimen is calculated. It is shown that the obtained strain distribution satisfies the unit-cell periodicity except at edges of the periodic structure, and that the strain distribution is in good agreement with the result of analysis using the homogenization theory. It is also shown that the macroscopic stress–strain relationships obtained by the experiment and analysis agree well, supporting the macroscopic and microscopic validity of the multiscale analysis using the homogenization theory.

1. Introduction

Multiscale analysis using the homogenization theory based on unit cell analysis [1,2] is regarded as one of the most powerful methods for investigating mechanical properties of heterogeneous materials with periodic internal structures. This analysis simultaneously provides the macroscopic properties of heterogeneous materials and microscopic fields of physical quantities in the materials. Moreover, since this analysis is performed by employing relatively small representative volume elements (unit cells) as analysis domains, it requires markedly lower computational costs compared with general finite element analysis in which finite element meshes of whole macroscopic domains are needed. Thus, the multiscale analysis has been used to study composites [3–12] as well as porous and cellar materials [13–16], some of which have been validated using experimental results [6–9,14,15].

However, nearly all these experimental validations were limited to macroscopic behaviors such as stress–strain relationships and creep curves. This limitation comes from the challenge of observing microscopic physical quantities in heterogeneous materials, especially in situ, in addition to their macroscopic behaviors. Thus, almost no studies perform microscopic validation of multiscale analysis of heterogeneous materials [6]. This gap in knowledge is important because the macroscopic validity of multiscale analysis does not guarantee its microscopic validity, and guaranteeing the quality and reliability of numerical analyses is becoming increasingly important [17–19]. It is therefore worth investigating the validity of multiscale analysis using the homogenization theory from a multiscale perspective.

In heterogeneous periodic materials, their microstructures generally consist of unit cells that are regularly aligned, but sometimes they are misaligned. For example, such misalignment can be found in plate-fin structures (Fig. 1) [20–24], commonly used in heat exchangers, as well as in the fabrics of woven composites [11,25–29]. In the present study, we refer to these materials with some misalignment as semi-periodic materials. Studying this issue is important when assuring the quality and reliability of multiscale analysis because real materials do not necessarily have perfectly aligned microstructures.

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http://dx.doi.org/10.1016/j.ijmecsci.2016.02.007
0020-7403/

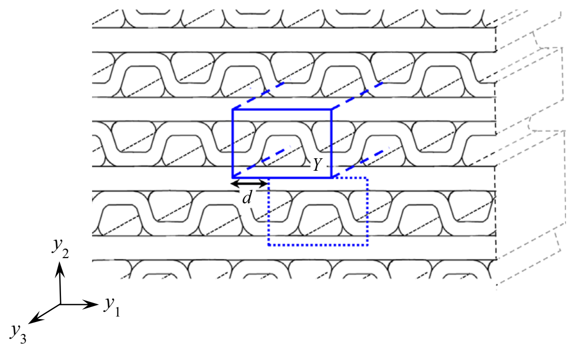

Fig. 1. Plate-fin-type semi-periodically perforated plate with misalignment d and unit cell Y.

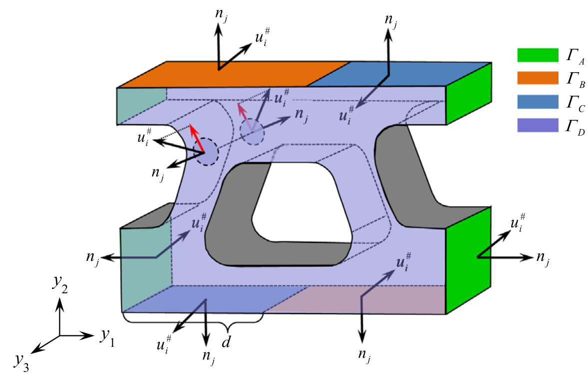

Fig. 2. Perturbed displacement $u_i^\#$ on the boundary edges of Y, Γ_A, Γ_B, Γ_C, and Γ_D.

Because of this reality, we have proposed a homogenization theory for semi-periodic materials [11], which enabled our multiscale homogenization analysis of semi-periodic materials with arbitrary misalignment by employing one unit cell as an analysis domain. Using this theory, we have analyzed the elastic-viscoplastic behavior of plain-woven fiber-reinforced composites with laminate misalignment of plain fabrics. Our results revealed that the macroscopic stress–strain relationship and the microscopic stress distribution of the composites depended much on the amount of misalignment. However, we could not experimentally validate these analysis results macroscopically and microscopically.

In recent years, full-field measurements based on digital image correlation (DIC) have been used to measure the displacement and strain fields on the material/structure surfaces [30,31]. This technique is able to measure displacement fields in arbitrary areas on the material/structure surfaces in situ by recording random patterns of sprayed ink on the surfaces and comparing those digital images before and after deformation. This technique has already been used to validate analytical solutions and finite element analyses [32–37]. An advantage of the DIC full-field measurement is that it can be applied to various scales: relatively large structures such as bridges and walls, moderate-size components such as tensile test specimens, and microscopic parts such as crack tips in metallic materials. A tensile testing machine combined with a DIC full-field measurement system might be able to simultaneously examine the macroscopic behavior and microscopic strain fields of periodic/semi-periodic materials, making it suitable for macro- and micro-validation of multiscale analysis using the homogenization theory.

In this study, multiscale analysis of a semi-periodic material based on a homogenization theory is experimentally validated both macroscopically and microscopically, using a DIC full-field strain measurement. For this, considering a plate-fin-type perforated plate with misaligned microstructure as a semi-periodic material, a homogenization theory for the perforated plate is presented. Then, a tensile test using a plate-fin-type semi-periodically perforated plate made of an epoxy resin is conducted at a constant macroscopic displacement rate. The macroscopic load is monitored by the tensile testing machine, while the microscopic strain field on the surface of the perforated plate is measured by a DIC full-field measurement system. Corresponding analysis is performed by the homogenization theory presented, and the macroscopic and microscopic validity of the multiscale analysis is examined by comparing the results of analysis and experiment.

2. Homogenization theory for semi-periodically perforated plates

Here, we consider a plate-fin-type semi-periodically perforated plate on the Cartesian coordinates y_i $(i=1,\ 2,\ 3)$, as shown in Fig. 1, with a misalignment d in the y_1-direction. The plate is assumed to have infinite length in the y_1- and y_2-directions. In contrast, it has finite length (plate thickness) in the y_3-direction. The plate is subjected to a macroscopically uniform load in the y_1–y_2 plane, and it responds with infinitesimal elastic deformation macroscopically and microscopically. Then, a unit cell Y for the structure is defined as shown in Fig. 1. In the following subsections, we derive a homogenization theory for the perforated plate by employing Y as the analysis domain.

2.1. Microscopic displacement and strain fields: mathematical modeling

We first introduce the microscopic displacement $u_i(\mathbf{y})$ in Y. From the macroscopic uniformity and the microscopic periodicity defined above, u_i should have the following expression:

$$u_i(\mathbf{y}) = (F_{ij} - \delta_{ij})y_j + u_i^\#(\mathbf{y}), \tag{1}$$

where F_{ij} is the macroscopic deformation gradient and $u_i^\#(\mathbf{y})$ is the perturbed displacement from the macroscopic one, $(F_{ij} - \delta_{ij})y_j$. In the y_1-direction, $u_i^\#$ distributes periodically in terms of Y, called the Y-periodicity. This causes the periodic distribution of $u_i^\#$ on the boundary edges of Y perpendicular to the y_1-direction (i.e., Γ_A), as illustrated in Fig. 2. However, with respect to the stacking direction, $u_i^\#$ distributes periodically not in the y_2-direction but in the direction inclined from the y_2-direction corresponding to the misalignment d [11]. This results in the periodic distribution of $u_i^\#$ on the boundary edges of Y perpendicular to the y_2-direction (i.e., Γ_B and Γ_C), as depicted in Fig. 2. We refer to this periodicity as M-periodicity (misaligned periodicity). In the y_3-direction, the periodicity of $u_i^\#$ is not defined because there is no repetition of Y in this direction, but the geometry of the perforated plate and the macroscopic uniformity allow $u_i^\#$ to distribute in mirror symmetry on Γ_D (Fig. 2).

Then, the microscopic strain $\varepsilon_{ij}(\mathbf{y})$ is expressed as a sum of the macroscopic strain E_{ij} and the perturbed strain $\varepsilon_{ij}^\#(\mathbf{y})$:

$$\varepsilon_{ij}(\mathbf{y}) = E_{ij} + \varepsilon_{ij}^\#(\mathbf{y}), \tag{2}$$

where E_{ij} and $\varepsilon_{ij}^\#$ satisfy the following relations:

$$E_{ij} = \frac{1}{2}(F_{ij} + F_{ji}) - \delta_{ij}, \tag{3}$$

$$\varepsilon_{ij}^\# = \frac{1}{2}(u_{i,j}^\# + u_{j,i}^\#). \tag{4}$$