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Hydroelastic dynamic analysis of axially moving plates in continuous hot-dip galvanizing process

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ABSTRACT

For the purpose of understanding the vibrational characteristics of moving plates in continuous hot-dip galvanizing process, the linear and nonlinear free vibrations of an axially moving rectangular plate coupled with dense fluid having a free surface are investigated. The fluid is assumed to be incompressible, inviscid and irrotational in this study. Effect of free surface waves of the fluid is taken into account in the analysis. The classical thin plate theory is adopted to formulate the equation of motion of the vibrating plate. The velocity potential and Bernoulli's equation are used to describe the fluid pressure acting on the moving plate. The effect of fluid on the vibrations of the plate may be equivalent to added mass of the plate. The system is solved by applying directly the method of multiple scales to the governing partial-differential equations and boundary conditions. Results show the immersion depth, moving speed, fluid-plate density ratio, stiffness ratio and aspect ratio all have significant effects on the natural frequencies of the immersed moving plate. The nonlinear frequencies of the plate-fluid system are influenced by initial amplitude, moving speed and nonlinear coefficient. It is also shown that the 1:1 and 1:3 internal resonances of the immersed moving plate can occur at certain speeds. Owing to the internal resonance, amplitude ratio of the two internal resonance modes shows multi-value characteristics. With the increase of nonlinear coefficient, the internal resonance phenomenon becomes more and more intense.

1. Introduction

Dynamic characteristics of axially moving continuum have interested many researchers in the recent 20 years. However, hydroelastic vibrations of axially moving continuum coupled with dense fluid are still scarce and the dynamical behaviour of this coupled system still need clarification.

The one-dimensional model for axially moving continuum can be found quite abundant in the literature. The literature review work on the dynamics of axially moving continuum in vacuo has been given in [1,2] and will not be repeated here. However, it is necessary to refer to some fundamental and some recent contributions. Briefly, nonlinear dynamics of one-mode approximation of an axially moving beam with varying speed was carried out by Ravindra and Zhu [3]. Öz and Pakdemirli [4] considered an Euler–Bernoulli beam having different flexural stiffness values and moving with harmonically varying velocities. They investigated principal parametric resonances and combination resonances of this model. By including stretching effect of the beam, Öz et al. [5] also analyzed the non-linear vibrations of an axially moving beam. Pellicano and Vestroni [6] investigated the bifurcation and stability of a simply supported axially moving beam subjected to an axial transport of mass. Then, they studied the dynamic response of this model subjected to a transverse load in the super-critical speed range [7]. Bifurcation and chaos dynamics were especially discussed in detail in this study. Riedel and Tan [8] studied the nonlinear response of an axially moving strip with coupled transverse and longitudinal motions. By using different mode expansions, Marynowski [9,10] investigated numerically the instability and bifurcation of axially moving viscoelastic beams. Yang and Chen [11] examined numerically bifurcation and chaos in transverse motions of accelerating viscoelastic beams with geometric nonlinearity. Employing the Timoshenko model, Yang et al. [12] investigates dynamic stability in transverse parametric vibrations of an axially accelerating tensioned beam on simple supports. The Galerkin method and the method of averaging were used in their study. Combining the multidimensional Lindstedt–Poincaré (MDLP) method and Galerkin method, Chen et al. [13] investigated the forced response of an axially moving beam with internal resonance between the first two transverse modes. Zhang and Song [14] studied higher-dimensional periodic and chaotic oscillations for a parametrically excited viscoelastic moving belt with

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Nomenclature

a	Length of the plate
b	Width of the plate
d	Height of fluid domain
D	Flexural rigidity of the plate
E	Young's modulus of the plate
h	Thickness of the plate
h_1	Fluid level below the plate surface
h_2	Fluid level on top of the plate surface
k	The nonlinear coefficient
m	Number of half-waves in the x-axis direction
M_x, M_y, M_{xy}	The internal moment resultants
n	Number of half-waves in the y-axis direction
N_0	Pretension per unit width in the axial direction
N_x, N_y, N_{xy}	The internal force resultants
p_L	Dynamic pressure on the lower fluid-plate interface
p_U	Dynamic pressure on the upper fluid-plate interface
t	Time
u, v, w	Displacements of mid-plane of the plate in the x, y, z direction, respectively
V	Axially moving speed of the plate
α_{mn}, β_{mn}	Amplitude and phase angle of the mnth mode
α_{mn_0}, β_{mn_0}	Initial amplitude and phase angle of the mnth mode
Δp	Dynamic pressure difference of the fluid
ε_x, ε_y, γ_{xy}	Normal strains in the x and y directions, as well as inplane shear strain at an arbitrary point of the plate, respectively
ε_x^0, ε_y^0, γ_{xy}^0	The middle-surface strains
ζ	The dimensionless stiffness ratio
μ	Poisson ratio of the plate
ξ	The dimensionless aspect ratio
ρ	The dimensionless density ratio
ρ_f	Mass density of the fluid
ρ_p	Mass density of the plate
σ_x, σ_y, τ_{xy}	Normal stresses in the x and y directions, as well as inplane shear stress, respectively
$\phi(x, y, z, t)$	The velocity potential function
χ_x, χ_y, χ_{xy}	Changes in the curvature and torsion of the middle surface in the corresponding coordinate directions.
ω_{mn}	The mnth linear natural frequency
Ω_{mn}	The mnth nonlinear frequency

multiple internal resonances. The parametric vibration and stability of an axially accelerating string guided by a partial non-linear elastic foundation was investigated analytically by Ghayesh [15]. Chen et al. [16] investigated the bifurcations and chaotic motions of higher-dimensional nonlinear systems for the nonplanar nonlinear vibrations of an axially accelerating moving viscoelastic beam. Based on Rayleigh beam theory, Chang et al. [17] delved into the vibration and stability of an axially moving beam by using finite element method. Ding and Chen [18] studied numerically the natural frequencies of planar vibration of axially moving beams in the supercritical ranges. Then they investigated steady-state periodical response for an axially moving viscoelastic beam with hybrid supports via approximate analysis with numerical confirmation [19]. Huang et al. [20] examined the nonlinear vibration of an axially moving beam subject to periodic lateral force excitations; they paid attention to the fundamental and subharmonic resonances and used the incremental harmonic balance method. The nonlinear vibration and control of an axially moving steel strip under aerodynamic excitations was carried out by Li et al. [21]. They examined the influences of variable production parameters on the vibration amplitude near the aerodynamic excitations in the study. Yao et al. [22] probed into the multi-pulse global bifurcations and chaotic dynamics for the nonlinear, non-planar oscillations of parametrically excited viscoelastic moving belts using an extended Melnikov method in the resonant case.

Compared with the one-dimensional model for axially moving continuum, researches using two-dimensional models are less. Based on the Mindlin–Reissner plate theory, Wang [23] developed a mixed finite element formulation for a moving orthotropic thin plate. By using the Kelvin-Voigt model, Zhou and Wang [24] discussed numerically the natural frequencies of axially moving viscoelastic rectangular plates with parabolically varying thickness. Hatami et al. [25] developed an exact finite strip method to analyze the free vibration of axially moving viscoelastic plates. Banichuk et al. [26] studied the loss of stability of axially moving plates in a two-dimensional formulation. The bending resistance and in-plane tension were taken into account. Tang and Chen [27] studied the natural frequencies, modes and critical speeds of in-plane moving rectangular plates on different supports. The complex natural frequencies for linear free vibrations and bifurcation and chaos for forced nonlinear vibration of an axially moving viscoelastic plate was investigated by Yang et al. [28]; the solution was obtained by using the finite difference method. Luo and Hamidzadeh [29] studied the buckling stability and post-buckling chaos of an axially moving plate by using Galerkin method and then numerical method. Based on von Kámán plate theory, Ghayesh et al. [30] investigated numerically the geometrically nonlinear vibrations and stability of an axially moving plate subjected to an out- of-plane excitation load. Based on Donnell's nonlinear shallow-shell theory, Wang et al. [31] introduced an improved nonlinear model to study the nonlinear dynamics of an axially moving thin circular cylindrical shell. They examined particularly the 1:1:1:1 internal resonance phenomenon of the structure.

All the works cited above considered the moving continuum in vacuum only, or, if there was fluid around the structure, it was ignored. However, it is known that the axially moving continuum always work in the dense fluid in some areas of engineering, such as the continuous hot-dip galvanizing process. Continuous hot-dip galvanizing process was first adopted by the United Kingdom in the 1930s, which now has been widely used in the production of auto sheet in the world. This process is shown in Fig. 1. In the continuous hot-dip galvanizing process, significant vibrations of the plate between touch rolls and stabilizing rolls, as seen in Fig. 1, can often occur. This worsens the local hot-dip galvanizing environment and results in the vibration stripe of the plate. In order to

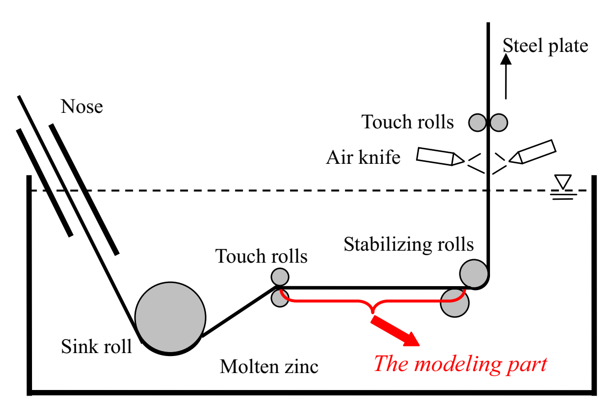

Fig. 1. Continuous hot-dip galvanizing process.