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An elastic approach for developing non-developable sheets

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ABSTRACT

Surface development origins from the cloth-making and computer graphics without consideration of the thickness, involving nonlinear optimization and constraints. Moreover the research of surface development mainly focuses on the planar development. In this paper, the development of the non-developable sheet to planar, singly-curved and doubly-curved surface patterns is investigated. An optimal developing algorithm is formulated to minimize the strain energy required for the deformation of the sheet, in which an orthogonal curvilinear coordinate system is used for three target patterns to simplify the constraints for the developing process, resulting in an unconstrained quadratic optimization. Both shell element and solid element are utilized in the finite element analysis. Similar developed results are obtained for planar and spherical patterns by using these two types of elements. But for the cylindrical pattern, the solid element model gives more accurate result due to no contribution of the shell element to the translation of the rotational freedom.

1. Introduction

Many engineering structures have three-dimensional (3D) surfaces that are fabricated from a planar, singly-curved, or even doubly-curved shape sheet. The first step of the fabrication in the process is the development of this 3D surface into a planar/singly-curved/doubly-curved shape so that the size of the initial shape (pattern) can be determined and the strain distribution required to form the shape can be estimated. 3D surfaces can be divided into two kinds of shapes, namely the developable surfaces and the non-developable surfaces. In general, the developable surfaces can be developed into a flat plate without membrane deformation, whereas the non-developable ones cannot be done without membrane deformation [1]. The non-developable surfaces (also called "doubly-curved surfaces") are widely used in cloth-making, ship-building and aerospace industries.

Current methods or algorithms for 3D freeform surfaces development are mainly based on the concept of optimized strain energy. Maillot et al. [2] established an energy model for flattening problem by polygonizing 3D surfaces and dealing it as a spring net, and then a minimization process was taken to reduce distortion during the surface development process. An energy-based flattening method can save both the material and energy needed in manufacture and avoid any unnecessary distortion in flattening process.

Yu et al. [3] presented a surface development approach in the help of differential geometry theory. They utilized the first and second fundamental form of the surface to express the shape before and after deformation. The approach described the problem as an optimization of the total strain energy with the constraint of zero Gaussian curvature after the deformation. Liu and Yao [4] used their algorithms to develop saddle and pillow shapes to a plane, in which the optimization problem involves multi-variables and multi-nonlinear constraints. However, the sophisticated mathematical description and its time-consuming computations limit its application. Moreover, the thickness is not taken into account in these methods.

Liang and Bin [5] came up with a flattening method, in which a subdivision was initially implemented to generate a discretized modal for a 3D surface and then all the nodes were forcibly unfolded onto a plane by unconstrained and constrained flattening methods. A spring-net model was adopted to approximately simulate the deformation energy and then an iterative process with the restrictions was employed to reduce total energy in flattening process. Liu et al. [6] employed an energy-based spring-mass model [7] and made some improvements in overlapping correction. The spring constant was assumed to be a constant for all springs in their model. Cai et al. [8] also introduced a planar flattening method for doubly-curved surface by minimizing the differences of each edge of the discretized elements before and

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Nomenclature

ε_α	Lamé coefficient along α direction
H_1	Lamé coefficient along α direction
ε_β	normal strain along β direction
H_2	Lamé coefficient along β direction
ε_γ	normal strain along γ direction
H_3	Lamé coefficient along γ direction
$\gamma_{\alpha\beta}$	shear strain in $\alpha\beta$ surface
A	value of H_1 at the pattern surface
$\gamma_{\beta\gamma s}$	hear strain in $\beta\gamma$ surface
B	value of H_2 at the pattern surface
$\gamma_{\gamma\alpha}$	shear strain in $\gamma\alpha$ surface
k_1	curvature of α lines at pattern surface
u, v, w	displacements in α, β, γ directions
k_2	curvature of β lines at pattern surface
V_α, V_β	rotation angle of shell element's nodes with respect to α and β axes
α_i, β_i, γ_i	coordinates of element's ith node in α, β, γ directions (in curvilinear coordinate system)
s, t, r	natural coordinates after isoparametric transformation
s_i, t_i, r_i	s, t, r directional coordinates of element's ith node (in natural coordinate system)
$\boldsymbol{\Upsilon}$	Poisson's ratio
E	Young's modulus
$\boldsymbol{J}$	Jacobian matrix of the isoparametric transformation
$\boldsymbol{K}$	element stiffness matrix
U_e	element's strain energy
U	total strain energy
$\boldsymbol{\Delta}$	freedom array of nodes
N	number of elements
N_i	shape function for ith node of the element
$\boldsymbol{D}$	constitutive matrix of sheet metal
$\boldsymbol{U}$	array of nodes' displacements
$\boldsymbol{B}$	strain matrix of element
R_c	radius of cylindrical pattern
R_s	radius of spherical pattern
γ_c	γ coordinate in cylindrical coordinate system
γ_s	γ coordinate in spherical coordinate system

after the development with necessary geometric constraints. This method reduces the 3D characteristic of original surface, which is different from many other methods used in literatures [5,6]. Also, this method takes the thickness of the sheet into account by considering the volume conservation of the element. However, the energy models in these studies are not exactly estimated because the ignorance of the shear stiffness and the hypothesis of constant spring constant which result in some errors in the energy estimation.

Ryu and Shin [1] adopted the classical elastic theory of shell with relatively large deformation to model the flattening process when using the minimization of total strain energy with nonlinear constraints. Different constraints were chosen for the cold and thermal forming of sheet metal. The thickness of sheet was also considered by dividing the strain into two parts: in-plane strain and out-of-plane strain, corresponding to membrane and bending strains respectively. However, the functions of optimization and constraints are very complicated and highly nonlinear. Cheng and Yao [9] analyzed the development of the doubly-curved surface using a commercial software package, which can save the computing time. They also proved that an elastic assumption is reasonable in developing process. However, only planar pattern is investigated in their work. In fact, the rolled sheets, which are singly-curved, are very common as initial blanks in the shipbuilding.

The literature survey shows that more refinements are still needed for surface development methods for sheets. Many current methods just regard flattening problem as a geographical problem because they origin from the fields like clothing [10,11] or computer graphics [12], in which the thickness of the surface is not taken into account. As the feasibility of energy-based developing models has been validated by many researches, energy-based approach is also adopted in this paper. Due to the inaccuracy in energy estimation of the spring-net model, FE method and elasticity theory are applied in this study to calculate strain energy and an optimization model is set up with geometrical constraints. Given the fact that rolled cylindrical plates are very common as the initial configuration for doubly-curved shapes in line heating of shipbuilding plates, developing processes from doubly-curved sheet to doubly-curved, singly-curved and planar pattern are of significance in practice. However, there is little research about developing from the doubly-curved shapes to singly-curved or doubly-curved shapes. Inspired by differential geometrical methods utilized in literature [3,4], this paper establishes a pattern-dependent orthogonal curvilinear coordinates system. In this way, complex constraint like zero Gaussian curvature can be avoided and geometrical constraints can be largely simplified. The present method is validated by developing a saddle sheet into planar, cylindrical and spherical patterns (Fig. 1) using both shell and solid elements.

2. Description of the problem

The doubly-curved surfaces are defined as the surfaces that have nonzero Gaussian curvature in at least some area. They have two nonzero principle curvatures and are not developable. In contrast, the singly-curved surfaces have only one nonzero principle curvature and they are developable. The planar surfaces are the surfaces with two zero principle curvatures. Because the development process of the doubly-curved shapes involves membrane deformation such as stretching and shrinking, the process is not unique. Thus, a principle must be set to determine

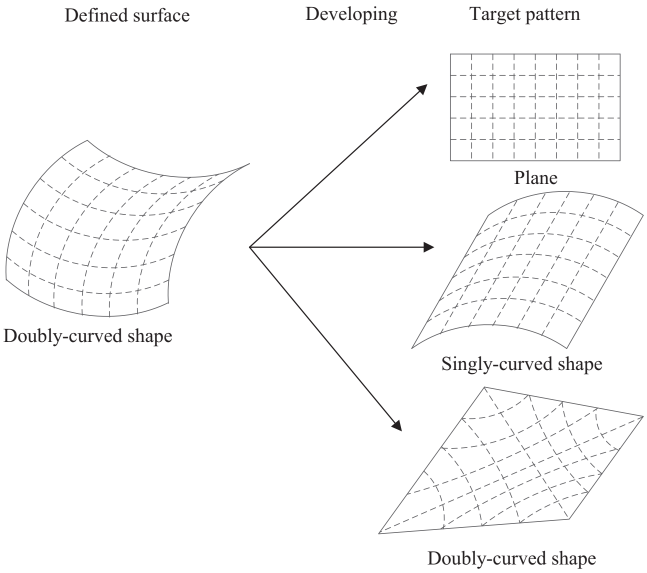

Fig. 1. Three kinds of target patterns in developing process for 3D freeform surface.