International Journal of Mechanical Sciences 106 (2016) 231–244

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Dispersion relations of elastic waves in one-dimensional piezoelectric phononic crystal with initial stresses

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ARTICLE INFO

Article history:
Received 16 September 2015
Received in revised form
26 November 2015
Accepted 18 December 2015
Available online 29 December 2015

Keywords:
Dispersion relations
Piezoelectricity
Phononic crystal
Initial stress
Bloch wave
Transfer matrix

ABSTRACT

The effects of initial stresses on dispersion relations of elastic waves in a one-dimensional piezoelectric phononic crystal are studied in this paper. It is taken into account that initial normal and shear stresses acted on piezoelectric slabs and their influences on constitutive equations, governing equations and boundary conditions under in-plane and anti-plane strain cases. Based on the transfer matrices of ingredient piezoelectric slabs, the total transfer matrix of a single cell is derived. Furthermore, the Bloch theorem is used to obtain the dispersive equation of in-plane and anti-plane Bloch waves. The dispersive equations are solved numerically and the numerical results are shown graphically. In the case of normal propagation of elastic waves within piezoelectric slabs, the analytical expressions of the dispersion equation are derived and compared with other literatures. The influences of initial stresses on the dispersive relations are discussed based on the numerical results. It is found that the initial normal stresses have more evident influences than that of initial shear stresses, no matter that in-plane Bloch waves or anti-plane Bloch wave are concerned. Moreover, the influences of initial stress are more evident on the dispersion curves at high frequency than that at the low frequency.

1. Introduction

Recently, the wave propagation through the finite and infinite one-dimensional laminated piezoelectric medium attracts the attention of many researchers [3]. studied the shear horizontal wave (SH wave) propagation based on a recursive system of equations involving the piezoelectric impedance in a layered piezoelectric composite [20]. also studied the propagation behavior of SH wave in a periodic layered structure. The layered structure consists of piezoelectric thin films bonded perfectly with polymeric thin films alternately. Filter effect of this kind of structure and the effects of volume fraction and shear modulus ratio of piezoelectric layer to polymeric layer on the velocity of SH wave are discussed in detail, respectively. The propagation of SH wave through the finite nonperiodic structures were studied by [17]. Two types of nonperiodic structures: Fibonacci sequences and systems with a linear perturbation in the piezoelectric parameters which give rise to resonances of Stark-Ladder type were considered. For the disordered layered piezoelectric composite structures [14], studied the localization of plane elastic waves and the expression of the localization factor in disordered structures was presented. The layered periodic structure considered by [19,18] includes not only the piezoelectric phases but also the piezomagnetic phases. The dispersive behavior and band gaps of elastic waves were investigated by the transfer matrix method [11]. studied shear waves propagating along the imperfectly bonded interface of a magnetoelectric composite consisting of Piezoelectric (PE) and Piezomagnetic (PM) phases. The dispersion relation and the existence condition of the interfacial shear waves were obtained [23]. studied the transmission behavior of shear horizontal (SH) wave propagating obliquely in layered structure formed by piezoelectric PZT and an Araldite polymer. The effects of the frequency, incident angle and piezoelectric volume fraction on the transmission behavior were discussed. In the investigations mentioned above, only the perfect interfaces were considered [12,13]. further studied the influence of mechanically imperfect interfaces and inhomogenous interlayers on the dispersive characteristics and the band gaps of SH waves [9]. studied not only the mechanically imperfect interface but also the dielectrically imperfect interfaces on the reflection and transmission waves at the interface of two piezoelectric half-spaces.

The existence of initial stresses in the layered structure is often inevitable due to mismatching thermal expansion, machining at different temperature, enhancing fracture toughness, creep deformation and chemical shrinkage and growth. In order to prevent the piezoelectric material from brittle fracture, the layered piezoelectric structure is usually pre-stressed during the manufacture process. The initial stress in the layered structures can lead

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http://dx.doi.org/10.1016/j.ijmecsci.2015.12.020
0020-7403/

to the dramatically change of dispersion relation. However, the studies above-mentioned are limited to the absence of initial stress. So it is necessary to investigate the influence of initial stress on propagation behavior of waves in such layered structure [5]. early developed a theory of incremental deformations superposed on initial stress within the geophysical context and he particularly studied the effect of initial stress on the propagation of small amplitude elastic waves [6]. studied the frequency equation and phase velocity of waves propagating in two kinds of laminated mediums under initial stresses [16]. investigated Bleustein–Gulyaev surface acoustic wave propagation in a prestressed layered piezoelectric structure. An almost linear behavior of the relative change in phase velocity versus the initial stress was found for both surface electrically free and shorted cases. The influence of initial stresses on the stop band and the dispersion relation were studied by [21]. The horizontally polarized shear waves in a periodic layered structure consisted of piezoelectric layer bonded perfectly with polymer layer alternately were considered. Their investigations showed that the stop band is dramatically changed with the increase of the magnitude of initial stress [22]. also investigated the propagation behavior of Love waves in a layered half-space with initial stress taken into account [25]. further studied the influences of the inhomogeneous initial stresses. It was concluded that the initial stress in layers affects the phase velocity, group velocity and electromechanical coupling coefficient. The elastic wave localization in disordered periodic piezoelectric rods with initial stress was studied by [26] based on the transfer matrix and Lyapunov exponent method. It was observed that the band structures can be tuned by exerting the suitable initial stress [10]. studied the dispersion equation of torsional surface waves in a homogeneous layer of finite thickness over an initially stressed heterogeneous half-space [8]. investigated the effects of initial stress on the reflection and transmission waves at the interface between two different piezoelectric half spaces. Compared with the theoretical investigation, the experimental investigations are less reported [7]. reported an experimental investigation about influences of initial stress on the band gaps and made the comparison with the theoretic predictions. Two kinds of finite phononic crystals with different initial stresses were investigated experimentally. It was observed that the initial stress can efficiently tune the location and width of the band gap. In general, the presence of initial stress makes a modification on the governing equation, constitutive equation and the boundary condition. However, the modification on the constitutive equation and boundary condition are often ignored in literature, e.g. [21,22,2,27,24,15]. Moreover, the initial stresses which are taken into consideration are often limited to the normal stress situation.

In this paper, the effects of initial stress on the dispersion relations of elastic waves in one-dimensional laminated structure composed of two different piezoelectric materials are studied. The normal initial stress and the shear initial stress are both considered for both in-plane and anti-plane strain cases. First, as initial stresses create finite deformations, we introduce the effective stress and the effective electric displacement to modify the constitutive equations, governing equations and boundary conditions considering the effects of initial stresses. Then, the total transfer matrix of one typical single cell in the periodical structure is obtained by the combination of the transfer matrices of two piezoelectric slabs. Finally, the Bloch theorem is used to obtain the dispersive equations of in-plane Bloch waves and anti-plane Bloch wave and the numerical results are shown graphically.

2. Constitutive and governing equations for a prestressed piezoelectric medium

The virtual work principle of piezoelectric solid in the finite strain situation can be expressed as [16,25]

$$\int_V (\boldsymbol{\sigma}:\delta\mathbf{S}-\mathbf{D}\cdot\delta\mathbf{E})\delta V-\int_V \rho_0(\mathbf{f}-\ddot{\mathbf{u}})\cdot\delta\mathbf{u}dV-\int_{\Sigma_m}\bar{\mathbf{t}}\cdot\delta\mathbf{u}d\Sigma$$
$$-\int_{\Sigma_e}\bar{q}\cdot\delta\varphi d\Sigma=0 \tag{1}$$

where $\boldsymbol{\sigma}$ is the second Piola-Kirchhoff stress tensor, $\mathbf{S}$ $(=0.5(\nabla\mathbf{u}+\mathbf{u}\nabla+\nabla\mathbf{u}\cdot\mathbf{u}\nabla))$ is Green strain tensor, ρ_0 is the mass density, $\mathbf{f}$ is the body force, $\mathbf{D}$ and $\mathbf{E}$ $(=-\nabla\varphi$, in the quasi-static field approximation) are the electric displacement and electric field vectors at the natural configuration, respectively. $\mathbf{u}$ is the displacement vector and φ is the electric potential. $\bar{\mathbf{t}}$ and $\bar{q}$ are the applied surface force and the electric charge density acting on per unit natural surface area. Σ_m and Σ_e are the boundary surfaces subjected to mechanical force and electric force, respectively. V is the volume enveloped by surface Σ_m and Σ_e. From the virtual work principle, the equation of motion, charge equation and associated boundary condition can be obtained.

$$(\sigma_{ik}F_{jk})_{,i}-\rho_0 f_j=\rho_0\ddot{u}_j \tag{2a}$$

$$D_{i,i}=0 \tag{2b}$$

$$\sigma_{ik}F_{jk}N_i=\bar{t}_j \tag{2c}$$

$$D_iN_i=\bar{q} \tag{2d}$$

where $F_{jk}(=\delta_{jk}+u_{j,k})$ is the displacement gradient tensor, and τ_{ij} $(=\sigma_{ik}F_{jk})$ is the first Piola-Kirchhoff stress tensor. N_i is the unit normal of natural boundary surface.

Consider a small incremental deformation (resulting from the elastic wave) superposing a finite deformation (resulting from the initial stress) in the piezoelectric medium. We can define the deformation state as the natural state (free stress state), the initial state (with initial stress) and the current state (with initial stress and incremental stress). Let

$$\sigma_{ij}^t=\sigma_{ij}^0+\sigma_{ij}, D_m^t=D_m^0+D_m, u_j^t=u_j^0+u_j, \varphi^t=\varphi^0+\varphi \tag{3}$$

where all the field quantities in the current state and initial state are denoted by superscript 't' and '0 ', respectively. σ_{ij}, D_i, u_i and φ are the incremental stress tensor, electric displacement and mechanical displacement vectors and electric potential resulting from elastic waves. Consider that Eq. (2) is applied to both the current state and initial state and that the initial state is static equilibrium state under initial stresses. By subtracting the equations corresponding with the initial state from that corresponding with the current state, we can obtain the governing equations for the incremental stress and the incremental electric displacement, and the associated boundary conditions [16,25]

$$\left(\sigma_{ij}+\sigma_{ik}u_{j,k}^0+\sigma_{ik}^0u_{j,k}\right)_{,i}=\rho\ddot{u}_j \tag{4a}$$

$$D_{m,m}=0 \tag{4b}$$

$$\left(\sigma_{ij}+\sigma_{ik}u_{j,k}^0+\sigma_{ik}^0u_{j,k}\right)N_i=\bar{t}_j \tag{4c}$$

$$D_iN_i=\bar{q} \tag{4d}$$

By comparing with the situation without initial stress, we can define the effective stress tensor and the effective electric