International Journal of Mechanical Sciences 106 (2016) 319–330

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Limit bending moment for pipes with two circumferential flaws under combined internal pressure and bending

Hao Ye [a], Jinyuan Qian [a], Sunting Yan [a], Chenghang Jiang [b], Lin Wei [a], Zhi-jiang Jin [a,*]

[a] *Institute of Process Equipment, Zhejiang University, Hangzhou 310027, China*
[b] *Zhejiang Academy of Safety Science and Technology, Hangzhou 310012, China*

ARTICLE INFO

Article history:
Received 18 August 2015
Received in revised form
8 December 2015
Accepted 26 December 2015
Available online 1 January 2016

Keywords:
Pipe
Circumferential flaws
Limit bending moment
Internal pressure

ABSTRACT

By assuming the neutral axis is perpendicular to the bending plane, this paper presents a general analytical solution to predict the limit bending moment for pipes with two inner surface circumferential flaws under combined internal pressure and bending moment. The circumferential stress caused by internal pressure is taken into account through Mises yield criterion. Based on the general solution, expressions for symmetrical and unsymmetrical bending with two flaws of the same size are proposed. Then the influence of perpendicularity assumption is discussed and it demonstrates that the deviation caused by perpendicularity assumption is negligible. Finite element analyses and experiments are both carried out. The results calculated by proposed expressions are found to be in good agreement with both numerical results and experimental ones. The effect of several parameters on the limit bending moment is also studied. Results reveal that circumferential stress caused by internal pressure has important influence on bending moment, and should be taken into account.

1. Introduction

Pipes are widely used in a variety of fields including petrochemical industry and nuclear power engineering, etc. Pipes will suffer complicated loads during operation and failure strength assessment of intact pipes and pipe bends has received widely attention (e.g. [1–6]). During in-service inspection of industrial pipes, circumferential flaws have been discovered, such as cracks and incomplete penetration flaws. These flaws jeopardize the safety operation of pipelines and may even lead to severe industrial accidents. Failure strength assessment of circumferential flawed pipes is also a very important subject with widespread application background.

Based on net-section collapse (NSC) criterion, Kanninen et al. [7,8] proposed solutions to predict plastic limit load for ductile pipes with circumferential flaws. This criterion assumes that plastic limit load is reached when the uniform net-section stress reaches flow stress. Later, Kastener et al. [9] derived a limit load solution for flawed cylinder based on the load to initial yield. Wilkowski and Eiber [10] developed an empirical expression to predict limit load for cylinders with through-wall flaw. Miller [11] compared results of the above methods with test results and J integrals calculated by finite element method, and concluded that the NSC criterion showed the best agreement with tests and J integrals. Besides, Limit load solutions for circumferential flawed pipe bends under pure bending are proposed by Chattopadhyay et al. [12,13].

The above analyses are limited to flaws with constant depth and symmetrical bending. Nevertheless, the actual flaws can take any shape and the idealization of constant depth will bring some conservatism. Rahman and Wilkowski [14,15] developed a general NSC method to predict limit load for cylinders with arbitrary-shaped circumferential flaws under combined tension and bending. Besides, flaws are not always symmetric with respect to bending plane and Rahman et al. [16] analyzed the effect of off-centered flaws on the crack-opening area under bending.

In the previous works only the equivalence of axial stress is considered. However, internal pressure will bring circumferential stress as well as axial stress. For pipes with shallow flaws, the ignorance of circumferential stress will overestimate the limit load [17]. After taking circumferential stress into consideration, Shu [18] gave plastic limit load solutions for pipes with off-centered circumferential flaws under bending, tension and internal pressure. Analytical solutions for thin-walled pipes with arbitrary-shaped circumferential flaws under combined symmetrical bending, tension and internal pressure are provided by Lei and Budden [19]. Jin et al. [20] derived a limit load solution for thin-walled flawed pipes under combined bending, torsion and internal pressure. These works assumed that limit load is reached when the Mises stress of every point on the net-section reaches flow stress.

* Corresponding author. Tel.: +86 571 8795 3099.
E-mail address: jzj@zju.edu.cn (Z.-j. Jin).

http://dx.doi.org/10.1016/j.ijmecsci.2015.12.024
0020-7403/

Nomenclature

a, a_1, a_2	flaw depths
D	outer diameter of pipe
E	Young's modulus
L	length of pipe
m_B	dimensionless limit bending moment
M_B	limit bending moment
M_{B0}	limit bending moment without flaws
Mx, My	x- and y- components of limit bending moment
n_θ	dimensionless circumferential stress
P	applied internal pressure
r	mean radius of pipe
R_o	outer radius of pipe
t	wall-thickness of pipe
x,y	Cartesian coordinates
α	half the angle between the two flaws
α_1, α_2	angles between the flaw and the bending plane
β_1,β_2	angles defining the position of neutral axis, with $\beta_1+\beta_2$ the central angle of neutral axis
β	angle defining the position of neutral axis under perpendicularity assumption
γ	angle between the bending plane and the symmetry plane of two flaws
η_1, η_2	dimensionless flaw depths
μ	Poisson ratio
θ, θ_1, θ_2	angles of flaws
θ_{11}, θ_{21}	angles of flaws in tensile stress zone
θ_{12}, θ_{22}	angles of flaws in compressive stress zone
σ_1, σ_2	tensile and compressive axial stresses at plastic limit state in the flawed section, $\sigma_1>\sigma_2$
σ_b	ultimate strength
σ_{eq}	equivalent stress
σ_f	flow stress
σ_y	yield strength
σ_θ	circumferential stress

For thick-walled pipes, the triaxial stress state must be considered. Limit load solutions for thick-walled flawed pipes have also been proposed (e.g. [21–26]).

Multiple circumferential flaws in the same cross section have been detected in pipes during inspection. To simply combine these flaws into a single one will inevitably get conservative limit load [27]. Although limit load solutions for pipes with single flaw have been widely analyzed, studies on failure behavior of pipes with multi-flaws are limited. Hasegawa et al. [27,28] developed equations for evaluating bending stress at collapse for pipes with multiple circumferential flaws. Machida et al. [29] also gave a plastic collapse assessment method for pipes with multi-flaws. However, in their studies the circumferential stress generated by internal pressure was not considered. Besides, the situation of unsymmetrical bending was not considered too.

This paper aims at proposing a more rational limit bending moment solution for pipes with two circumferential flaws under combined internal pressure and bending moment. The effect of circumferential stress is taken into consideration in this paper. By assuming the neutral axis is perpendicular to the bending plane, general expressions are first proposed. Then expressions without this perpendicularity assumption are also derived and the influence of this assumption is quantified. The validity of the general expressions is verified by finite element analyses as well as experiments. The proposed solution will be used to study the influence on the limit bending moment of parameters such as flaw depth, circumferential length and circumferential stress.

2. Limit bending moment under combined internal pressure and bending

The following assumptions are used in deriving the solution: (1) the pipe is thin-walled pipe, and the radial stress could be ignored, the axial stress and circumferential stress will be constant along the radial direction; (2) the cross section remains circular during loading.

According to NSC criterion, when pipe reaches the plastic limit state, the stress in net section is uniform-distributed and equals to the flow stress σ_f. When the pipe is under combined internal pressure and bending moment, the section with flaws is under biaxial stress state, so it can be considered that the Mises equivalent stress σ_{eq} equals to the flow stress σ_f,

$$\sigma_{eq}=\sqrt{\frac{1}{2}[(\sigma-\sigma_\theta)^2+\sigma^2+\sigma_\theta^2]}=\sigma_f \tag{1}$$

where σ_θ is the circumferential stress. Define the dimensionless circumferential stress $n_\theta=\sigma_\theta/\sigma_f=Pr/(t\sigma_f)$, where P is the applied internal pressure, r is the mean radius of pipe and t is wall-thickness of pipe. Substituting this into Eq. (1), the section normal stress σ can be calculated as

$$\frac{\sigma_{1,2}}{\sigma_f}=\frac{n_\theta\pm\sqrt{4-3n_\theta^2}}{2} \tag{2}$$

where σ_1 is the tensile stress above the neutral axis in the section with flaws and σ_2 is the compressive stress below the neutral axis. Flow stress σ_f is given by the average of yield stress and ultimate tensile stress.

2.1. Two arbitrary circumferential flaws

Generally, the two flaws are in arbitrary positions with different sizes. In order to evaluate the limit bending moment for pipes with two arbitrary circumferential flaws, a solving method based on the actual sizes and positions of flaws is proposed.

Fig. 1 illustrates the schematic of the pipe cross-section containing two circumferential flaws. The bending moment M_B is on the y-axis plane. Since the two flaws are unsymmetrical, the actual neutral axis will not be perpendicular to the applied plane of bending moment ($\beta_1\neq\beta_2$, where β_1 and β_2 are angles defining the

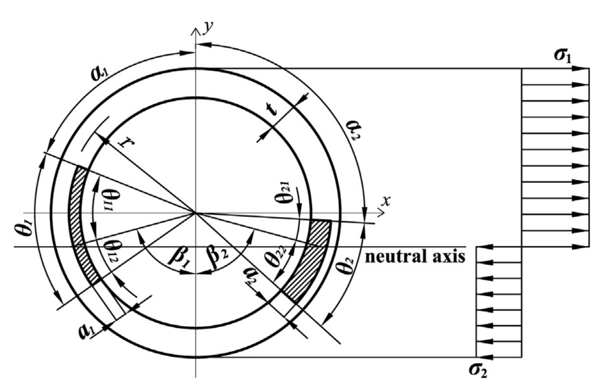

Fig. 1. Schematic and stress distribution of pipes with two arbitrary circumferential flaws.