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# Non-linear nonlocal vibration and stability analysis of axially moving nanoscale beams with time-dependent velocity

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ABSTRACT

The extraordinary properties of carbon nanotubes enable a variety of applications such as axially moving elements in nanoscale systems. For vibration analysis of axially moving nanoscale beams with time-dependent velocity, the small-scale effects could make considerable changes in the vibration behavior. In this research, by applying the nonlocal theory and considering small fluctuations in the axial velocity, the stability and non-linear vibrations of an axially moving nanoscale visco-elastic Rayleigh beam are studied. It is assumed that the non-linearity is geometric and is due to the axial stress changes. The energy loss in the system is considered by using the Kelvin–Voigt model. The governing higher order nonlocal equation of motion is derived by using Hamilton's principle and is analyzed by applying the multiple scales and power series methods. Then the non-linear resonance frequencies and response of the system are obtained. Considering the solvability condition, the stability of the system is studied parametrically through Lyapunov's first method. An interesting result is that, considering the small-scale effects changes the slope of the frequency response curves due to the fluctuations in the axial velocity, considerably.



## 1. Introduction

Due to the outstanding mechanical and physical properties of the carbon nanotubes, wide range of applications in different areas of nanotechnology exists. So, an extensive amount of studies have been done on the dynamics and stability of the systems in the nanoscale, such as fluid conveying carbon nanotubes [1–6]. In these studies the effect of fluid velocity on the vibration and stability of carbon nanotubes is studied. Because of the variety of the carbon nanotubes' applications, there is a need for further investigations on the related fields. Carbon nanotubes are expected to become unstable at higher axial velocities because of their high elastic modulus to the density ratio. Some potential applications of axially moving nanoscale beams would be in spacecraft antennas, space elevator cables, flexible nanorobotic manipulators, high speed vehicles and conveyors of nanoscale belts. For these usages, investigations on the vibration of the axially moving nanoscale beams are of high importance.

According to the macroscale applications of axially moving beams, several studies have been done on the vibration of such systems. These studies indicate that the axial velocity plays a great role in the evolution of the researches conducted, because variable or constant velocity, changes the elastic behavior of the beam. The dynamics of these systems is studied in two sub- and super-critical axial velocity regimes, the critical velocity is the limit between these two regimes, in which instability occurs. On the other hand, axial velocity fluctuations could cause instabilities even in a very low axial velocity [7–13]. Pakdemirli and Ulsoy [14] studied the parametric principal resonance and combination resonance for any two modes of axially moving string. They introduced a velocity function having small harmonic fluctuations about a constant mean velocity, this velocity better indicates many real systems. They showed that for velocity fluctuation frequencies close to twice any natural frequencies, an instability occurs, however for the velocity fluctuation frequencies close to zero no instability happens. Ghayesh and Khadem [15] studied the effects of the rotary inertia and temperature on the non-linear vibration and stability regions of the steady-state response of an axially moving beam with time-dependent velocity. They showed that increasing the rotary inertia, natural frequencies and critical velocities reduce. In another similar study, Ghayesh and Balar [11] investigated the effects of the geometric and mechanical parameters on the vibration behavior, non-linear resonance frequencies and stability regions of an axially moving beam. With further study on the technical literature concerning these systems, one can find that a wide range of research works has been done on the vibration and stability of axially moving beams in the macroscale.

In the nanoscale, the molecular dynamic simulation could be an appropriate method to study the vibration of systems, but this method requires an enormous computation effort. So a continuum molecular model is essential to study nanotechnology related

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problems. Nonlocal elasticity theory is one of the most important models for studying nanoscale systems [16,17]. Based on this theory, the scale effect is taken into account and in contrast with classic models, stress at a point is a function of strain at all points of the body. Lim et al. [18] studied the dynamic behavior of an axially moving nanobeam with a constant speed. They investigated the nonlocality effects on the natural frequencies and critical velocities. Kiani [19] studied the longitudinal, transverse, and torsional vibrations and stabilities of axially moving single-walled carbon nanotubes with a constant speed. He used the nonlocal elasticity theory and discretized the governing equations of motions based on the Galerkin method. The discretization is done by using the assumed mode-shape functions corresponding the mode-shapes of the beam with zero axial velocity.

All the mentioned studies show that the instability due to the axial velocity fluctuations, non-linear vibration and mode-shapes are not investigated in the nanoscale. In this investigation, small-scale effects on the vibration behavior and stability regions of a nanoscale beam with a time-dependent axial velocity are studied. By directly applying the multiple scales method to the higher order equation of motion and using a power series method, natural frequencies, complex mode-shapes and response of the beam are derived. Then by using solvability condition, the stability of the steady-state response is studied.

## 2. The equation of motion

As shown in Fig. 1, consider an axially moving nanoscale beam with time-dependent velocity $v(t)$. The beam has the length $l$, mass density $\rho$, cross sectional area $A$, cross-sectional area moment of inertia $J$, Young's modulus $E$ and initial tension $P$.

The kinetic energy based on Rayleigh beam model is given by [13]

$$T(t)=\frac{1}{2}\int_0^l \rho A\left(w_{,t}+vw_{,x}\right)^2 \mathrm{d}x+\frac{1}{2}\int_0^l \rho J\left(w_{,xt}+vw_{,xx}\right)^2 \mathrm{d}x \tag{1}$$

where $w(x,t)$ is the displacement in the $z$ direction. In order to take into account the geometric non-linearity due to large amplitude, the Lagrangian strain is used as [20]

$$\varepsilon_x=\frac{1}{2}w_{,x}^2-zw_{,xx} \tag{2}$$

The nonlocal beam theory is used to consider the small-scale effects. This theory is an agreement between the atomic theory of lattice dynamics and experimental observations. According to this theory, the stress at a point of a body is dependent on the strain at all the points in the body. For a homogenous isotropic material, Eringen proposed the following equation as the nonlocal stress field [16]:

$$\sigma_x^{nl}-(e_0a)^2\sigma_{x,xx}^{nl}=\sigma_x^l \tag{3}$$

where $\sigma_x^l$ is the local axial stress, $\sigma_x^{nl}$ is the nonlocal axial stress, $e_0$ is the material constant determined experimentally and $a$ is the characteristic length. The value of $e_0a$ should be calibrated by using the molecular simulation results. This parameter is a function of the boundary conditions and molecular lattice [21,22]. Resultant nonlocal bending moment and axial force are defined as

$$M^{nl}=-\int_A z\sigma_x^{nl}\mathrm{d}A, \tag{4}$$

$$N^{nl}=\int_A \sigma_x^{nl}\mathrm{d}A \tag{5}$$

Assuming the energy loss in the system based on the Kelvin–Voigt model with viscosity coefficient $\eta$, considering the strain to

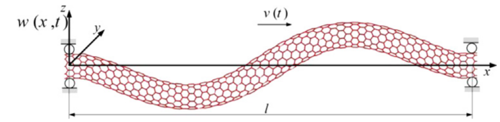


**Fig. 1.** Axially moving nanoscale beam with both ends hinged.

be a function of both time and space, and using Eq. (3)–(5), the resultant nonlocal bending moment and axial force are obtained as the following implicit forms:

$$M^{nl}-(e_0a)^2M_{,xx}^{nl}=EJw_{,xx}+\eta J\left(w_{,xxt}+vw_{,xxx}\right) \tag{6}$$

$$N^{nl}-(e_0a)^2N_{,xx}^{nl}=\frac{1}{2}EAw_{,x}^2+\eta A\left(w_{,xt}w_{,x}+vw_{,x}w_{,xx}\right) \tag{7}$$

These equations are obtained to derive new nonlocal equation of motion. The potential energy due to the bending moment and the axial force is

$$U(t)=\frac{1}{2}\int_0^l M^{nl}w_{,xx}\mathrm{d}x+\frac{1}{4}\int_0^l\left(N^{nl}+P\right)w_{,x}^2\mathrm{d}x \tag{8}$$

Using Hamilton's principle, the following equation of motion can be derived:

$$\rho A\left(w_{,tt}+2vw_{,xt}+\dot{v}w_{,x}+v^2w_{,xx}\right)$$
$$-\rho J\left(w_{,xxtt}+\dot{v}w_{,xxx}+2vw_{,xxxt}+v^2w_{,xxxx}\right)+M_{,xx}^{nl}-\left(N^{nl}w_{,x}+Pw_{,x}\right)_{,x}=0 \tag{9}$$

In this research, the longitudinal vibration is ignored. So, for the axial force one can obtain

$$N_{,xx}^{nl}=0 \tag{10}$$

Using Eqs. (6), (7), (9) and (10), and defining the following dimensionless parameters:

$$w^*=\frac{w}{l}, \tag{11}$$

$$x^*=\frac{x}{l}, \tag{12}$$

$$t^*=t\sqrt{\frac{P}{\rho Al^2}}, \tag{13}$$

$$v^*=v\sqrt{\frac{\rho A}{P}}, \tag{14}$$

$$\tau=\frac{e_0a}{l} \tag{15}$$

$$v_f=\sqrt{\frac{EJ}{Pl^2}}, \tag{16}$$

$$v_1=\sqrt{\frac{EA}{P}} \tag{17}$$

$$\gamma=\frac{\eta J}{Pl^3}\sqrt{\frac{P}{\rho A}}, \tag{18}$$

$$\zeta=\frac{\eta A}{PL}\sqrt{\frac{P}{\rho A}}, \tag{19}$$