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Multi-transform based spectral element to include first order shear deformation in plates

M.V.V.S. Murthy [a], K. Renji [a,*], S. Gopalakrishnan [b]

[a] Strucutres Group, ISRO Satellite Center (ISAC), Indian Space Research Organization, Vimanapura Post, Bangalore 560017, India
[b] Department of Aerospace Engineering, Indian Institute of Science, Bangalore 560012, India

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ABSTRACT

For obtaining dynamic response of structure to high frequency shock excitation spectral elements have several advantages over conventional methods. At higher frequencies transverse shear and rotary inertia have a predominant role. These are represented by the First order Shear Deformation Theory (FSDT). But not much work is reported on spectral elements with FSDT. This work presents a new spectral element based on the FSDT/Mindlin Plate Theory which is essential for wave propagation analysis of sandwich plates. Multi-transformation method is used to solve the coupled partial differential equations, i.e., Laplace transforms for temporal approximation and wavelet transforms for spatial approximation. The formulation takes into account the axial-flexure and shear coupling. The ability of the element to represent different modes of wave motion is demonstrated. Impact on the derived wave motion characteristics in the absence of the developed spectral element is discussed. The transient response using the formulated element is validated by the results obtained using Finite Element Method (FEM) which needs significant computational effort. Experimental results are provided which confirms the need to having the developed spectral element for the high frequency response of structures.

1. Introduction

Honeycomb sandwich panels are widely used in aerospace structures. They are constructed with face sheets bonded to a compliant core which is light. The higher stiffness to weight ratio of these structures enables to carry higher payloads. These structures are subjected to high frequency dynamics excitation, especially shock. Design of such structures needs a suitable model to predict its behaviour to high frequency dynamic excitation.

Typical load bearing honeycomb sandwich structures are normally modelled as beams and plates. Plate structures with $kh < 1$ is typically analysed based on the Kirchoff Plate Theory/Classical Plate Theory (CPT), where 'k' is the wavenumber and 'h' is the thickness of the plate. When the bending wavelength is comparable with the thickness of the plate, the effect of shear deformation and rotary inertia needs to be considered [1]. It is known that transverse shear effects play a vital role in the transmission characteristics of the waves at shorter wavelengths. As honeycomb sandwich construction uses a core, whose transverse shear flexibility is very high compared to a solid structure, the influence of transverse shear deformation is expected to be significant. Comparison of dispersion curves obtained with exact (3D formulation of theory of elasticity) and simplified theories (2D formulation as generalization of the Timoshenko theory) concluded [2] that this simplified theory can be reliably used to assess the waveguide properties of sandwich plate in the frequency range of interest. Liu and Bhattacharya [3] dealt with wave propagation in infinite sandwich structure and gave the complete description of dispersion relation without any restriction on frequency and wavelength. On the other hand, a vast number of displacement based theories exist in the literature for analysis of sandwich structures. Ref. [4] presented a sixth order theory for damped sandwich beams and Ref. [5] presented a higher order theory for capturing the local behaviour of the core caused by the flexibility in transverse direction, which is essential at high frequencies. A review of displacement based theories and the finite element models thereof are presented in [6]. Based on the above, the First order Shear Deformation Theory (FSDT)/Mindlin Plate Theory (MPT), which represents the shear deformation effects accurately, with a suitable shear correction factor, is a relatively simple tool and has been found to yield accurate results for sandwich structures.

It is very common that the dynamic analysis of structures with finite dimensions is usually approached by Finite Element Method (FEM). However, it becomes computationally expensive to capture the responses in high frequency limits due to the reduced element sizes required for shorter wavelengths. Spectral analysis which is the synthesis of waveforms from superposition of many frequency components [7] takes us to the realm of frequency domain analysis. Spectral analysis in the frequency domain using matrix methodology, called as Spectral Element Method (SEM) [8], is a

* Corresponding author. Tel.: +91 80 25083602; fax: +91 80 25083603.
E-mail address: renji@isac.gov.in (K. Renji).

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very useful technique to determine the response to dynamic excitation at high frequencies. In the spectral element method, the size of the element does not rely on the wavelength and thus caters to both short as well as long wave propagation problems.

There is abundant literature in the frequency domain SEM addressing 1D (rods and beams) problems. Ref. [9] presents the majority of the works in this field. On the contrary, only a few have addressed 2D plate problems in the spectral element domain. The dynamic stiffness matrix for the 2D Kirchoff plate is presented by the continuous element method in [10] where Gorman's method of boundary condition decomposition and Levy's series are used to obtain the strong solution. Also the solution to the strong form of the 2D Kirchoff plate governing equations by SEM is presented in [8] for an isotropic plate. Here, the in-plane and out-of-plane decoupled equations have been solved in the Fourier domain. SEM for Levy type isotropic plates has been developed in [11]. Here, as before in the Fourier domain, the 2D problem is transformed to an equivalent 1D problem, by determining the horizontal in-plane displacement function in analytical form, so as to satisfy the simply supported boundary conditions, along the two parallel sides (i.e., across the width for the plate). As for orthotropic plates based on the Kirchoff Plate Theory, the coupled equations have been solved in the Fourier domain and is presented in [12]. Later, the same problem has been approached by the wavelet transform method [13]. Though there are several works on spectral element formulation of Kirchoff plates, spectral element for plates considering transverse shear deformation is not discussed. In the previous works on spectral element for plates, wavelet transform has been applied to both the temporal and the spatial component of the field variable. For the temporal component, the wavelet transform removed the problems of wrap around associated with the Fourier transform, but it is observed that the wavenumber and group speeds are accurate only up-to a certain fraction of the Nyquist (central frequency). If the analysis is performed beyond this frequency, one can see non-existent spurious dispersion [14]. Therefore the Laplace transformation is preferred for the temporal variable [15].

In this work, a 2D SEM considering transverse shear deformation is developed. For incorporating transverse shear deformation Mindlin's Plate Theory, which is the FSDT, is used. One of the important parameters in using Mindlin's Plate Theory is the shear correction factor. In this work the value of shear correction factor applicable to sandwich structures [16] is incorporated. Numerical Laplace transform for the temporal part is used to eliminate the signal wrap around. The real part of the Laplace transform acts as numerical damping to the solution and forces the response to decay down within the chosen time window [17] whereas the wavelet transform is used for one of the spatial component. The unique feature of this work is that two sets of transformations are used. Such an approach, which has not been attempted so far, combines the advantage of both the transforms and thus removing the signal wrap around completely. This is the first multi-transformation spectral element to be reported in the literature. Spectrum and dispersion relations are obtained for typical structures using the developed element and its ability to represent various modes of wave motion is demonstrated. Need for considering the transverse shear deformation is highlighted. Validation of the spectral element with the results of FEM is made. Finally the results obtained by the spectral element are compared with the experimental results for a typical sandwich plate to validate the need for such spectral elements.

2. Equations of wave motion in thick plates

First order Shear Deformation Theory (FSDT), which is based on the Mindlin Plate Theory (MPT) [18], is used to represent the transverse shear effects.

2.1. Strains, stresses and forces

Consider a rectangular plate as shown in Fig. 1(a). The displacement field as per FSDT is

$$U(x,y,z,t) = u(x,y,t) + z\theta_x(x,y,t) \tag{1a}$$

$$V(x,y,z,t) = v(x,y,t) + z\theta_y(x,y,t) \tag{1b}$$

$$W(x,y,z,t) = w(x,y,t) \tag{1c}$$

where u, v and w are the displacement components of the reference plane in x, y, and z directions respectively and θ_x and θ_y are the rotations of the transverse normal about the y and x axes respectively (Fig. 1(b)). The associated strains are

$$\begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{Bmatrix} + z \begin{Bmatrix} \frac{\partial \theta_x}{\partial x} \\ \frac{\partial \theta_y}{\partial y} \\ \frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_x}{\partial x} \end{Bmatrix} = \{\epsilon_0\} + z\{\epsilon_1\} \tag{2a}$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial w}{\partial y} + \theta_y \\ \frac{\partial w}{\partial x} + \theta_x \end{Bmatrix} \tag{2b}$$

where ϵ_{xx} and ϵ_{yy} are the normal strains in x and y directions respectively. γ_{xy} is the in-plane shear strain and γ_{yz} and γ_{xz} are the transverse shear strains.

Assuming special orthotropy the normal and shear stresses are related to these above strains through the constitutive relations:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & 0 \\ \overline{Q}_{12} & \overline{Q}_{22} & 0 \\ 0 & 0 & \overline{Q}_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} \tag{3a}$$

$$\begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{44} & 0 \\ 0 & \overline{Q}_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{3b}$$

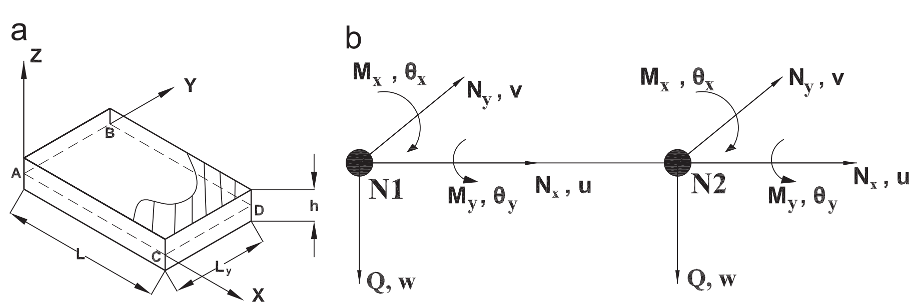

Fig. 1. (a) Plate geometry and (b) geometric and force boundary conditions.