International Journal of Mechanical Sciences 96-97 (2015) 163–171

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Application of the first-order shear deformation theory to the analysis of laminated glasses and photovoltaic panels

J. Eisenträger*, K. Naumenko, H. Altenbach, H. Köppe

Otto von Guericke University Magdeburg, Universitätsplatz 2, 39106 Magdeburg, Germany

ARTICLE INFO

Article history:
Received 12 December 2014
Received in revised form
2 March 2015
Accepted 13 March 2015
Available online 21 March 2015

Keywords:
Photovoltaic panel
First-order shear deformation theory
FEM
User element

ABSTRACT

Laminated plates and photovoltaic panels are composed of three layers, whereas the core layer, comprising the solar cells and their encapsulation, is more shear-compliant than the skin layers. First-order shear deformation theories (FSDT) like the Mindlin theory are usually applied to these laminated plates in combination with a homogenisation approach. If the differences in shear stiffnesses are too strong, the FSDT may fail to predict the deformation behaviour accurately. This paper evaluates the applicability range of FSDT to the laminated glass and photovoltaic panels. Furthermore, a user-defined element is integrated with the subroutine *User Element* in Abaqus. This element utilises a homogenisation approach to determine the effective material parameters. To verify the results of the finite element analysis, a closed-form series solution is applied. The attention is placed on the accurate representation of the boundary layer effects that are important for the strength analysis.

1. Introduction

Laminated plates and shells with skin layers from glass and a core layer from Polyvinyl Butyral (PVB) are widely used in the civil engineering and automotive industry [1–3]. Crystalline or thin film photovoltaic modules currently available on the market are composed from front and back glass or polymer layers and a solar cell layer embedded in a polymeric encapsulant [4–6], cf. Figs. 1 and 2. Various materials like Ethylene Vinyl Acetate (EVA) and PVB are applied to encapsulate the solar cells [4]. In lightweight variants of photovoltaic modules, the front and back plates are made from plastics. These skin layers are connected together by a transparent Polyurethane (PUR), in which the solar cells are embedded [7], cf. Fig 3.

During the operation, laminated glass plates and solar modules are subjected to thermo-mechanical loadings, for example snow or wind loads and daily or seasonal temperature changes. For the design of a laminate, it is beneficial to analyse the suitability of materials like PVB, EVA, or PUR for core layer or for embedding solar cells. These encapsulates have to compensate different mechanical and thermal strains of bottom and top layers. In order to predict stress and strain states in the core layer, structural analyses of the laminate under the thermo-mechanical loadings are required.

One feature of laminated glass plates or laminates used in photovoltaic industry is the layered composite with relatively stiff skin layers and relatively thin and compliant polymer encapsulant layer. Let G_S be the shear modulus of the glass skin layer and G_C the shear modulus of the polymeric core layer. The ratio of the shear moduli $\mu = G_C/G_S$ for materials used in photovoltaic panels is in the range between 10^{-5} and 10^{-2}, depending on the type of polymer and the temperature [4,7,8]. For the comparison, classical sandwich panels are composed from materials with μ in the range of 10^{-2} and 10^{-1}. In addition, in classical sandwich structures the face sheets are thin in comparison with the core, while in photovoltaic applications the face layers are relatively thick and the core is relatively thin.

Various structural mechanics models are available to analyse the behaviour of laminated glass and photovoltaic panels. A widely used approach for sandwich and laminate structures is the first-order shear deformation theory (FSDT) [9,10]. This theory is based on the assumption that the normals to the midsurface of the plate behave like rigid bodies during the deformation. The local mechanical interactions between cross-sections are characterised by forces and moments. The advantage of this theory is the possibility to solve the governing differential equations in a closed analytical form for plates of various shapes and boundary conditions. Closed-form solutions or approximate analytical solutions for plates according to the FSDT are presented in [9–14] among others. Furthermore, plate or shell elements available in standard finite element codes are usually based on FSDT, e.g. [15]. A key step in the application of the FSDT is to estimate effective characteristics of the layered system, in particular the properties related to the transverse shear deformation. Closed-form relationships are developed to find effective elastic stiffness of a

* Corresponding author.
E-mail address: johanna.eisentraeger@ovgu.de (J. Eisenträger).

http://dx.doi.org/10.1016/j.ijmecsci.2015.03.012
0020-7403/

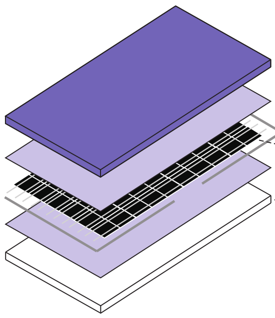

Fig. 1. Crystalline solar module [4].

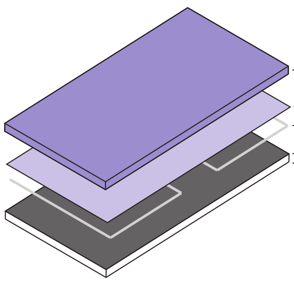

Fig. 2. Thin film solar module [4].

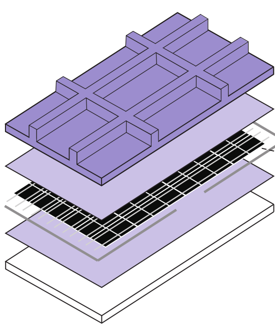

Fig. 3. Lightweight solar module, after [7].

laminate from the properties of the layers, e.g. [4]. To estimate the effective transverse shear deformation in the inelastic range, various numerical techniques are available [16,17].

Laminated glasses and photovoltaic panels can also be analysed by the use of the three-dimensional theory of elasticity and applying the finite element method for the numerical solution. To this end, various types of continuum shell finite elements and three-dimensional solid finite elements are available in commercial codes, e.g. [15]. Due to extreme differences in material properties of the constituents and the relatively low thickness of the core layer, considerable numerical effort is required to obtain the results with a desired accuracy [4,18]. In particular, care should be taken for finite element meshing the core layer in order to compute the transverse shear strains and the related stresses accurately.

Recently, layer-wise theories have been developed and applied in order to analyse laminated structures. Within the layer-wise theory (LWT), balance and constitutive equations are derived for individual layers. With constitutive assumptions for interaction forces and compatibility conditions, a model for the layered system is derived. For laminates with core layer from soft polymers, LWT are presented in [1–4,7] for beams and in [19,20] for plates. To derive the robust equations, the assumption is made that glass skin layers deform according to the Bernoulli–Euler assumptions for beams or Kirchhoff assumptions for plates. The soft core layer carries out the transverse shear stresses only, while the bending moments and the membrane forces are neglected. In [1,4,7], results of three point bending tests for beams with core layers from various polymers are presented. Closed-form solutions derived with the LWT agree well with the experimental data. Furthermore, as shown in [3,4,7], the solutions according to the LWT agree well with the results of the three-dimensional finite element analysis. However, closed-form or semi-analytical solutions to equations of LWT are presented only for simply supported plates [19] and plate strips with various boundary conditions [20] and the corresponding finite elements are not available in standard finite element codes.

The aim of this paper is to analyse the applicability of the FSDT to laminated glasses and photovoltaic panels. To this end, we address the following problems:

- For laminates with extreme differences in the stiffness properties of the constituents, the FSDT may fail to predict the deformation properties of the laminate. This is demonstrated in [4,7] for beams. For plates, a shear rigidity parameter should be introduced to capture the validity range of the FSDT for glass and photovoltaic panels. The values of this parameter must be evaluated for EVA at different temperature levels.
- For laminates with soft core, accurate representation of boundary layer effects is crucial for the strength analysis. In particular, transverse shear stress and transverse shear deformation should be predicted accurately in the vicinity of the plate boundaries. To this end, results of the numerical analysis by the finite element method must be compared with the closed-form solutions.

2. First-order shear deformation theory

Governing equations: This paragraph introduces the governing equations of the FSDT using a tensor notation. Tensors are represented by bold upper-case letters, whereas bold lower-case letters are used for vectors. Greek letters used for indices take the values 1 or 2. The Einstein summation convention is applied if indices appear twice in one term. In this paper, we apply the direct tensor calculus in the sense of Gibbs [21] and Lagally [22]. Within this calculus, a second-rank tensor is a finite sum of dyads of vectors, for example $\boldsymbol{A} = \boldsymbol{a} \otimes \boldsymbol{b} + \boldsymbol{c} \otimes \boldsymbol{d} + \cdots$. In analogy, a fourth-rank tensor $\mathbb{A} = \boldsymbol{a} \otimes \boldsymbol{b} \otimes \boldsymbol{c} \otimes \boldsymbol{d} + \boldsymbol{e} \otimes \boldsymbol{f} \otimes \boldsymbol{g} \otimes \boldsymbol{h} + \cdots$ is a finite sum of tetrads of vectors. In the following, basic operations for dyads and tetrads are introduced:

$$\boldsymbol{a} \otimes \boldsymbol{b} \cdot \boldsymbol{c} = \alpha \boldsymbol{a}, \quad \alpha = \boldsymbol{b} \cdot \boldsymbol{c} \tag{1}$$

$$\boldsymbol{c} \cdot \boldsymbol{a} \otimes \boldsymbol{b} = \beta \boldsymbol{b}, \quad \beta = \boldsymbol{c} \cdot \boldsymbol{a} \tag{2}$$

$$\boldsymbol{a} \otimes \boldsymbol{b} \cdot\cdot\, \boldsymbol{c} \otimes \boldsymbol{d} = \alpha\beta, \quad \alpha = \boldsymbol{b} \cdot \boldsymbol{c}, \quad \beta = \boldsymbol{a} \cdot \boldsymbol{d} \tag{3}$$

$$\boldsymbol{a} \otimes \boldsymbol{b} \otimes \boldsymbol{c} \otimes \boldsymbol{d} \cdot\cdot\, \boldsymbol{e} \otimes \boldsymbol{f} = \alpha\beta \boldsymbol{a} \otimes \boldsymbol{b}, \quad \alpha = \boldsymbol{d} \cdot \boldsymbol{e}, \quad \beta = \boldsymbol{c} \cdot \boldsymbol{f} \tag{4}$$

Operations (1)–(4) are generalised for tensors and used in many textbooks on continuum mechanics and rheology, see for example [23–25].

Fig. 4 shows a rectangular plate subjected to the uniform surface load $\boldsymbol{q} = q\boldsymbol{n}$. The coordinate system with the orthonormal basis $\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{n}$ and the corresponding coordinates x_1, x_2, z is applied. The position vector with respect to the current state $\boldsymbol{r}$ of the points belonging to the plate midsurface is defined as follows:

$$\boldsymbol{r} = \boldsymbol{R} + \boldsymbol{u} + w\boldsymbol{n}, \tag{5}$$

where $\boldsymbol{R} = x_\alpha \boldsymbol{e}_\alpha$ is the position vector in the reference state, $\boldsymbol{u}$ is the in-plane displacement vector and w is the deflection. The cross-section rotations can be characterised by the vector $\boldsymbol{\Theta} = \Theta_\alpha \boldsymbol{e}_\alpha$. The components Θ_α are illustrated in Fig. 4. Instead