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Analytic free vibration solutions of rectangular thin plates point-supported at a corner

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ABSTRACT

In a recently published study, we reported the benchmark bending solutions of rectangular thin plates point-supported at a corner by an up-to-date symplectic superposition method. In this paper, for the first time, we extend the approach to the free vibration problems of the same plates and obtain the analytic solutions which cannot be obtained by the conventional symplectic approach. More than three hundred frequencies and the representative mode shapes are presented as the benchmarks for verification of various new approaches developed in future.

1. Introduction

The static problems of the rectangular thin plates point-supported at a corner have been analytically solved by the symplectic superposition method in 2014 [1]. The boundary conditions of the two opposite edges to the point-supported corner could be any combinations of clamped and simply supported conditions, as shown in Fig. 1 (a)–(c) where the distributed transverse load q is applied on the plates in the Cartesian coordinate system (*x and y*). At that stage, the free vibration solutions were not attainable due to our limited understanding that there were much physical difference between the vibration problems and the static ones. However, we recently found that, within the framework of the Hamiltonian system and symplectic space [2], the description of free vibration of a plate is mathematically similar to bending and the former could be handled using the same method as that for the latter.

In recent years, Lim et al. [3] developed the analytic solutions for free vibration of Lévy-type rectangular thin plates by the symplectic elasticity approach and found that, except for vibration problems [4], the approach has very high potential for analytic plate bending solutions such as those of the corner-supported rectangular thin plates [5]. Actually, the symplectic approach has been applied in many research fields, including elasticity [2,6], symplectic numerical methods [7–9], fracture mechanics [10], piezoelectricity [11], functionally graded effects [12], magneto-electro-elasticity [13], stretchable electronics [14], etc. For further details, the reader is referred to the excellent review article by Lim et al. [15], which systematically introduces the fundamental theory and various applications of the approach.

It should be noted that the conventional symplectic approach does not exhibit the ability to analytically solve the vibration of rectangular plates with free edges while without two opposite edges simply supported. Actually, the most recent advance in analytic modeling of the plates' vibration is the application of a new method of separation of variables to plates with only clamped and simply supported edges [16].

In this paper, we further develop the symplectic superposition method so that it is applicable to the free vibration problems. The plates with the same boundary conditions as those in Ref. [1] are investigated, whose analytic solutions have not been reported in the literature as far as we know. The method is not semi-inverse, and can be applied to any combinations of classical boundary conditions (clamped, free, simply supported, and slidingly clamped). The cases with a corner point support are focused on in the following.

2. Free vibration of a thin plate within the framework of the Hamiltonian system

Compared with the static problem [1], the circular frequency ω enters the functional Π_H of the Hamiltonian variational principle

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for the free vibration of a thin plate, as shown in Fig. 1(d)–(f) corresponding to the static problems in Fig. 1(a)–(c), so that

$$\Pi_H = \iint_\Omega \left[\frac{D}{2}\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \frac{D}{2}\left(\frac{\partial \theta}{\partial y}\right)^2 + D\nu\frac{\partial^2 w}{\partial x^2}\frac{\partial \theta}{\partial y} + D(1-\nu)\left(\frac{\partial \theta}{\partial x}\right)^2 - \frac{D}{2(1-\nu^2)}\left(\frac{M_y}{D} + \frac{\partial \theta}{\partial y} + \nu\frac{\partial^2 w}{\partial x^2}\right)^2 + T\left(\theta - \frac{\partial w}{\partial y}\right) - \frac{\rho h\omega^2}{2}w^2\right] dxdy \tag{1}$$

where, Ω denotes the plate domain in the Cartesian coordinate system (x *and* y); w is the plate lateral displacement; M_y is the bending moment; ν is the Poisson's ratio; D is the flexural rigidity; ρ is the mass density; h is the plate thickness; T is the Lagrange multiplier; θ is defined as $\partial w/\partial y$.

With the independent variables w, θ, T, and M_y, $\delta\Pi_H = 0$ gives the governing matrix equation

$$\partial \mathbf{Z}/\partial y = \mathbf{HZ} \tag{2}$$

where, $\mathbf{Z} = [w, \theta, T, M_y]^{\mathrm{T}}$,

$$\mathbf{H} = \begin{bmatrix} \mathbf{F} & \mathbf{G} \\ \mathbf{Q} & -\mathbf{F}^{\mathrm{T}} \end{bmatrix},$$

$$\mathbf{Q} = \begin{bmatrix} \rho h\omega^2 - D(1-\nu^2)\partial^4/\partial x^4 & 0 \\ 0 & 2D(1-\nu)\partial^2/\partial x^2 \end{bmatrix},$$

$$\mathbf{F} = \begin{bmatrix} 0 & 1 \\ -\nu(\partial^2/\partial x^2) & 0 \end{bmatrix},$$

$$\mathbf{G} = \begin{bmatrix} 0 & 0 \\ 0 & -1/D \end{bmatrix},$$

$\theta = \partial w/\partial y$, and $T = -V_y$ in which V_y is the equivalent shear force. The Hamiltonian operator matrix [2] $\mathbf{H}$ satisfies $\mathbf{H}^{\mathrm{T}} = \mathbf{JHJ}$, where,

$$\mathbf{J} = \begin{bmatrix} 0 & \mathbf{I}_2 \\ -\mathbf{I}_2 & 0 \end{bmatrix}$$

is the symplectic matrix in which $\mathbf{I}_2$ is the 2×2 unit matrix.

It is obvious that Eq. (2) is homogeneous because there is no external load applied, which is different from the static problem where the load is indispensible. Another major difference is that ω must be determined before w is obtained in free vibration; that is, Eq. (2), with appropriate boundary conditions, cannot be solved unless ω is known. When using the conventional symplectic approach for the vibration of plates without two opposite edges simply supported, one would get stuck in the step toward the determination of ω because a transcendental equation with two undetermined constants, the eigenvalue and the frequency, is inevitable; and this is the reason why there has been little progress on the subject.

In the following, we extend the recently developed symplectic superposition method to rectangular thin plates' free vibration, and obtain the analytic solutions of the plates point-supported at a corner for the first time. This extension is valuable in view of the ability of the method to yield benchmark analytic solutions of the plates without two opposite edges simply supported in a rigorous manner without predetermining the solution forms.

3. Analytic free vibration solutions of the plates point-supported at a corner by the symplectic superposition method

3.1. Solutions of the subproblems

The basic idea is that the title problem is regarded as the superposition of two subproblems which can be rationally solved by the symplectic approach, as shown in Fig. 2(a) for a plate point-supported at a corner and clamped at its opposite edges. The first subproblem (Fig. 2(b)) is the vibration of a plate with the edges at $x = 0$ and $x = a$ simply supported, and with the displacement $w|_{y=0} = \sum_{n=1,2,3,\ldots}^{\infty} E_n \sin(\alpha_n x)$ and bending moment $M_y|_{y=b} = \sum_{n=1,2,3,\ldots}^{\infty} F_n \sin(\alpha_n x)$ distributed along the simply supported edges at $y = 0$ and $y = b$, respectively, where E_n and F_n are the coefficients of Fourier series expansion to be determined. The second subproblem (Fig. 2(c)) is the vibration of the plate with the edges at $y = 0$ and $y = b$ simply supported, and with the displacement $w|_{x=0} = \sum_{n=1,2,3,\ldots}^{\infty} G_n \sin(\beta_n y)$ and bending moment $M_x|_{x=a} = \sum_{n=1,2,3,\ldots}^{\infty} H_n \sin(\beta_n y)$ distributed along the simply

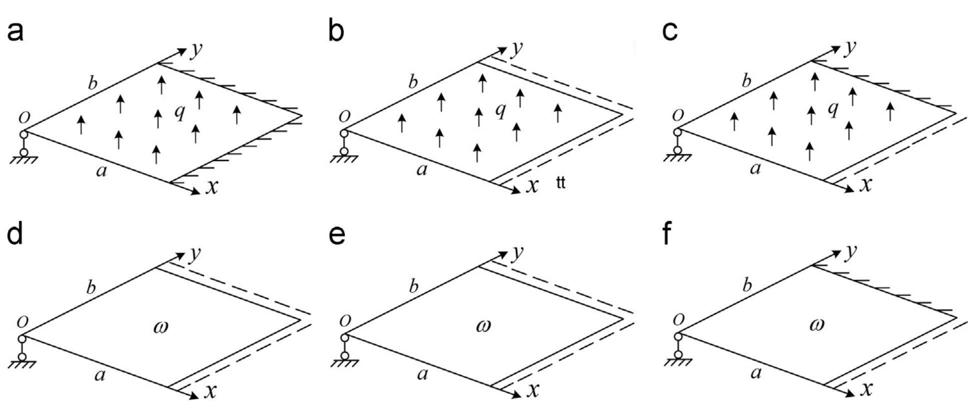

Fig. 1. (a)–(c) Static problems of rectangular thin plates point-supported at a corner and clamped, or simply supported at its opposite edges, or clamped at one of its opposite edges and simply supported at the other one. (d)–(f) Free vibration problems corresponding to the plates (a)–(c).

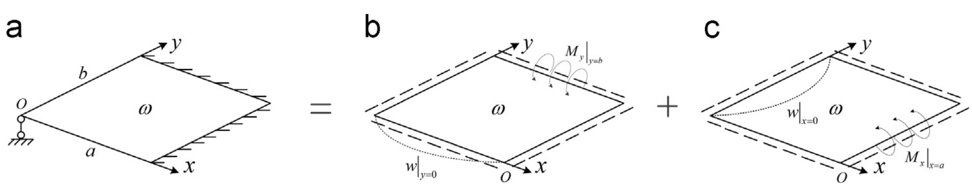

Fig. 2. Symplectic superposition for the free vibration of a rectangular thin plate point-supported at a corner and clamped at its opposite edges.