ARTICLE IN PRESS

International Journal of Mechanical Sciences ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Impact and rebound of an elastic–plastic ring on a rigid target

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ARTICLE INFO

Article history:
Received 10 March 2013
Received in revised form
26 January 2014
Accepted 27 March 2014

Keywords:
Thin-walled ring
Free impact
Rebound
Coefficient of restitution

ABSTRACT

This paper studies the impact and rebound behaviour of a thin-walled elastic–plastic circular ring after it impinges onto a rigid wall with initial velocity V_0 by finite element method. Through the dimensional analysis and a systematic simulation with different ring geometries and material properties, three non-dimensional parameters are identified, which dominate the impact duration and rebound velocity of the ring; and they are: (1) the ratio of wall thickness to average radius, $\eta = h/R$; (2) the yield strain of the material, Y/E; and (3) the non-dimensional initial velocity of the ring, $\nu \equiv V_0/V_Y$, where $V_Y \equiv Y/(E\rho)^{1/2}$ denotes the yield velocity of the material [3].

When the initial velocity is low, the impact between the ring and rigid plate remains elastic, the compression duration could be analytically obtained, and it agrees well with the numerical results, while the restitution duration is about 3/4 of the compression duration. The corresponding coefficient of restitution (COR) is found to be independent from the material property, whereas it increases from 0.75 to 0.78 when the thickness ratio changes from 1/20 to 1/40. With the increasing of initial velocity, a four-hinge crushing mode and a subsequent five-hinge mode are identified, which are different from the crushing mode of a ring under static compression. The variations of compression and restitution durations, rebound velocity and COR are discussed in detail, while some of them are compared with those of the impact of a solid ball onto a rigid wall. The effects of the ring geometry and material properties on these variables are also presented. The rebound velocity is found to reach the maximal at about a half of material's yield velocity when the initial velocity of the ring is about twice of the yield velocity, resulting in COR being 0.25 for all the materials and geometries adopted in this paper.

1. Introduction

As a simple but fundamental structural member, a circular ring has attracted much attention in applied mechanics over the last 50 years. In studying the quasi-static crushing of a rigid-plastic ring between rigid targets, de Runtz and Hodge [1] first proposed a 4-hinge mechanism in 1963. In 1964, Yu studied a rigid-plastic circular ring pulled by a pair of point loads along diameter direction, as published in Chinese [2] and then summarized in a book in English [3]. With a systematic study over the period of 1970s–1980s (e.g. [4–7]), Professor Steve Reid made significant contributions to the comprehensive understanding on the behavior of plastic circular rings under quasi-static and dynamic loading.

For a rigid-plastic free circular ring subjected to a suddenly applied force, a 4-hinge mechanism was also found [8], in which the side hinges' positions depend on the magnitude of the force applied. Recently, based on a study of an elastic ring on an elastic foundation (RoF) [9], Zhou et al. [10] investigated the dynamic response of an elastic ring on visco-elastic foundation to external impact, exploring that the dynamic deformation modes of the RoF system are governed by elastic bending rigidity ratio between the ring and foundation, as well as the latter's viscosity.

There are a plenty of free-bodies' impact problems in engineering applications and our daily life, such as car crashing on wall, aircraft's and spacecraft's landing, sports ball's rebound from ground, and collision of granular materials etc. In those situations, impact duration, deformation mode, energy transformation and energy absorption, rebound behaviour and thereafter coefficient of restitution (COR) are the most important characteristics to be concerned. Numerous papers [11–18] have studied the impact and rebound of solid bodies, but there is a lack of the research on the thin-walled structures. The present paper is aimed to study the dynamic response and rebound behavior of a thin-walled free-flying circular ring made of elastic, perfectly plastic material impinging onto a stationary rigid wall with initial velocity V_0, and explore the differences between thin-walled structures and solid bodies in impact and rebound. A systematic numerical simulation

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http://dx.doi.org/10.1016/j.ijmecsci.2014.03.031

is conducted to reveal the dynamic crushing modes, the impact duration, the velocity of the ring after it rebounds from the rigid plate and the corresponding COR, etc. The governing parameters will be identified with the analysis of systematic simulation results, and the effects of ring geometry and material properties will be presented. The results will be useful for us to understand the dynamic deformation mechanisms of such a thin-walled structural member after it impinges to a rigid plate and the following COR.

2. Finite element model

Fig. 1 shows the schematic drawing of the models, where Fig. 1(a) illustrates a thin-walled circular ring compressed by two parallel rigid plates and Fig. 1(b) depicts a thin-walled circular ring impinging freely onto a stationary rigid wall with initial velocity V_0. The basic geometry parameters of the ring are chosen as follows: the averaged radius of the ring $R=0.1$ m, the ratio of wall thickness to radius $\eta \equiv h/R=1/20$, and the ratio of ring width to radius $b/R=1/20$ that ensures the plane-stress assumption [3]. The ring is assumed to be made of aluminum alloy, and the basic material properties are: elastic modulus $E=70.0$ GPa, Poisson's ratio $\mu=0.3$, mass density $\rho=2700$ kg/m^3 and yield stress $Y=150.0$ MPa. However, to essentially reveal the impact and rebound behaviour of the ring, a systematical simulation is

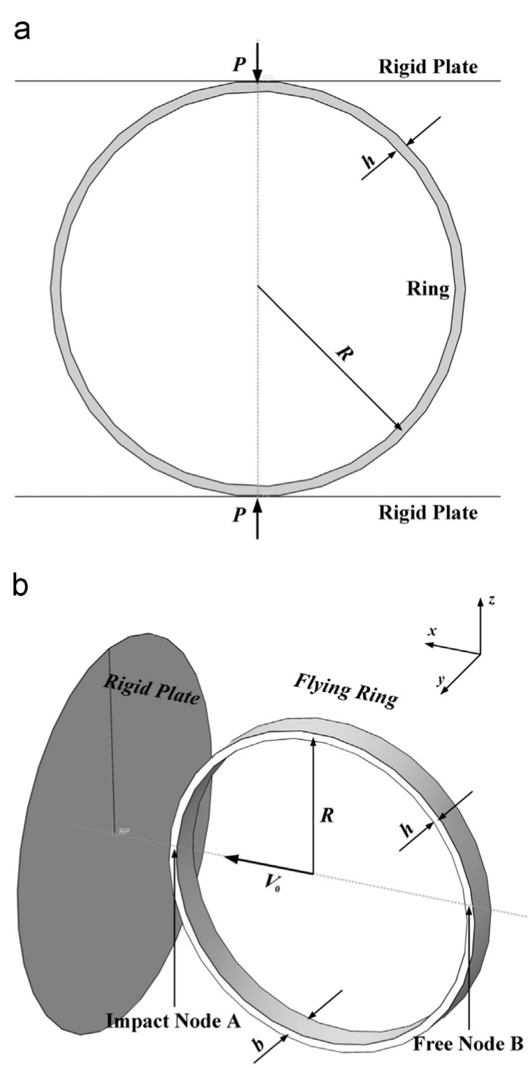

Fig. 1. Scheme of the FEM models: (a) a ring compressed by two parallel rigid plates and (b) a freely flying ring impinges onto a stationary rigid plate.

necessary to cover a wide range of geometry and material properties, including R, η, b/R, E and ρ.

From a dimensional analysis, it is evident that one of the governing parameters dominating the dynamic performance of the ring is $\nu \equiv V_0/V_Y$, where $V_Y \equiv Y/(E\rho)^{1/2}$ is the yield velocity of the material [3]. With preliminary FEM simulation results, it is found that the non-dimensional impact velocity ν should vary in the range from 0.05 to 3.5 to ensure that all the crushing modes can occur during the impact of the ring.

ABAQUS/Standard and ABAQUS/Explicit modules are employed in this study to simulate the static compression and dynamic impact of the ring, respectively. Due to the symmetry of geometry as schematically shown in Fig. 1, only a half of the ring along the y- and z-directions as shown in Fig. 1(b) is adopted in the finite element model, whilst appropriate symmetry boundary conditions are imposed. The wall of the ring is discretized by using shell elements S3 (3-node triangular general-purpose shell) and S4R (4-node general-purpose shell with finite membrane strains, reduced integration and hourglass control) [19], while five integration points are adopted through the shell thickness for bending. The mesh size is chosen to be less than the 1/60 of the radius R, and there are at least 6 elements along the width (i.e., the y-directions as shown in Fig. 1(b)) for a half ring. The interaction between the ring and the rigid wall is modeled as a surface-to-surface contact pair with "hard" contact interaction property [19]. According to the preliminary simulations, the friction coefficient of the contact pair does not significantly influence the final results, so that all the contacts are assumed to be frictionless in this study for the sake of simplicity. A restart analysis [19] is adopted in ABAQUS/Explicit to detect the impact duration between the ring and the rigid target, which is unknown prior to the simulation.

3. Static crushing of the ring

When a thin-walled ring is laterally compressed by two parallel rigid plates as shown in Fig. 1(a), Timoshenko [20] derived an elastic solution of the load provided it is a plane stress problem.

$$P=\frac{Eh^3b}{12R^3}\left(\frac{\pi}{4}-\frac{2}{\pi}\right)\delta \quad (1)$$

where P is the load applied on the plates and δ is the shortening in the distance between two plates. On the other hand, when a rigid, perfectly plastic ring is crushed by two parallel rigid plates, de Runtz and Hodge [1] proposed a four-hinge mechanism, and obtained the crushing load as

$$P=\frac{P_0}{\sqrt{1-(\delta/D)^2}} \quad (2)$$

where $P_0=Yh^2b/R$ is the initial collapse load and $D=2R$ is the diameter of the ring. Reddy [4] further improved the solution by de Runtz and Hodge to include the effect of strain-hardening.

Fig. 2(a) compares the elastic solution of Eq. (1) and the fully plastic solution of Eq. (2) with the finite element simulation results, where $E=70$ GPa, $Y=150$ MPa, $R=0.1$ m, $\eta=1/20$, and $b/R=1/2$, 1/5, 1/10 and 1/20, respectively. It can be seen clearly from Fig. 2(a) that the simulation results agree very well with Eq. (1) and the width of the ring has negligible effect within the elastic deformation regime. However, when the non-dimensional deflection δ/D reaches beyond 0.05, the effect of the ring width can no longer be ignored. With the decreasing of ring width, the discrepancy between FEM result and the perfectly plastic solution provided by Eq. (2) becomes more and more insignificant. Thus, by taking $b/R=1/20$, the FEM model can be approximated as a plane stress problem.

Please cite this article as: Bao RH, Yu TX. Impact and rebound of an elastic–plastic ring on a rigid target. Int. J. Mech. Sci. (2014), http://dx.doi.org/10.1016/j.ijmecsci.2014.03.031