International Journal of Mechanical Sciences ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Nonlinear dynamic Interactions between flow-induced galloping and shell-like buckling

J. Michael T. Thompson [a,b,*], Jan Sieber [c]

[a] Centre for Applied Dynamics Research, School of Engineering, University of Aberdeen, AB24 3UE, UK
[b] DAMTP, Cambridge, UK
[c] College of Engineering, Mathematics and Physical Sciences, Harrison Building, University of Exeter, Exeter EX4 4QF, UK

ARTICLE INFO

Article history:
Received 29 January 2013
Received in revised form
6 December 2013
Accepted 16 December 2013

Keywords:
Nonlinear dynamics
Interactions
Galloping
Shell-like buckling

ABSTRACT

For an elastic system that is non-conservative but autonomous, subjected for example to time-independent loading by a steadily flowing fluid (air or water), a dangerous bifurcation, such as a sub-critical bifurcation, or a cyclic fold, will trigger a dynamic jump to one or more remote stable attractors. When there is more than one candidate attractor, the one onto which the structure settles can then be *indeterminate*, being sensitive to infinitesimally small variations in starting conditions or parameters.

In this paper we develop and study an archetypal model to explore the nonlinear dynamic interactions between galloping at an incipient sub-critical Hopf bifurcation of a structure with shell-like buckling behaviour that is gravity-loaded to approach a sub-critical pitch-fork bifurcation. For the fluid forces, we draw on the aerodynamic coefficients determined experimentally by Novak for the flow around a bluff body of rectangular cross-section. Meanwhile, for the structural component, we consider a variant of the propped-cantilever model that is widely used to illustrate the sub-critical pitch-fork: within this model a symmetry-breaking imperfection makes the behaviour generic.

The compound bifurcation corresponding to simultaneous galloping and buckling is the so-called Takens-Bodganov Cusp. We make a full unfolding of this codimension-3 bifurcation for our archetypal model to explore the adjacent phase-space topologies and their indeterminacies.

1. Introduction

The simplest form of pure galloping is exhibited by a bluff body oscillating transversely in a steady wind. With a structural support providing both linear elastic stiffness and linear viscous damping, the theory for this phenomenon was developed by Novak [1] for a series of rectangular cross-sections. Based on experimental fitting to the quasi-static aerodynamic forces, Novak's theory agreed well with his related experimental studies. An excellent modern account of this, and other work, is given in the book by Paidousis et al. [2]. Note that galloping is essentially a one-mode phenomenon, distinct from flutter which arises in systems with at least two active modes; and even more distinct from vortex resonance which involves a strong interaction with the fluid. Note, though, that in nonlinear dynamics the bifurcations to both galloping and flutter are described as a Hopf bifurcation [3–5].

The essence of Novak's galloping theory was to use the highly nonlinear aerodynamic force characteristics obtained by calibration experiments in which a steady wind-stream was directed, at a series of (resultant) angles, towards the stationary rectangular body. The characteristic graph of lateral force versus angle of attack was then approximated by a seventh-order polynomial. Some of Novak's results are summarised in Fig. 1. Here the lateral force on the rectangular prism, in the direction of the lateral displacement, x, due to a wind of velocity, V, is ½ $\rho a V^2 C_f(\alpha)$ where ρ is the air density, a is the frontal area, and the (small) angle α is approximately x'/V. A prime denotes differentiation with respect to the time, t. The responses in the right-hand column show the amplitude of the steady-state oscillations. These periodic motions are stable when represented by a solid line, unstable when represented by a broken line. Hopf bifurcations on the trivial solution are denoted by H, and away from the trivial path stable and unstable oscillatory regimes meet at cyclic folds. Fast dynamic jumps are indicated by vertical arrows.

For case (a) the wavy arrow denotes a slightly turbulent wind (elsewhere the wind is steady). The 2:1 rectangular cross-section exhibits a super-critical Hopf bifurcation at H, with a path of stable limit cycles for higher values of the wind speed. In row (b) the square cross-section in a steady wind exhibits at H a super-critical Hopf bifurcation; and the subsequent limit cycles exhibit two cyclic folds and an associated hysteresis cycle. In row (c) a 2:1 rectangle in steady wind exhibits a sub-critical Hopf bifurcation at H from which a fast dynamic jump would carry the system

* Corresponding author at: DAMTP, University of Cambridge, Cambridge, CB30WA, UK.
E-mail address: jmtt@ucl.ac.uk (J.M.T. Thompson).

0020-7403/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ijmecsci.2013.12.007

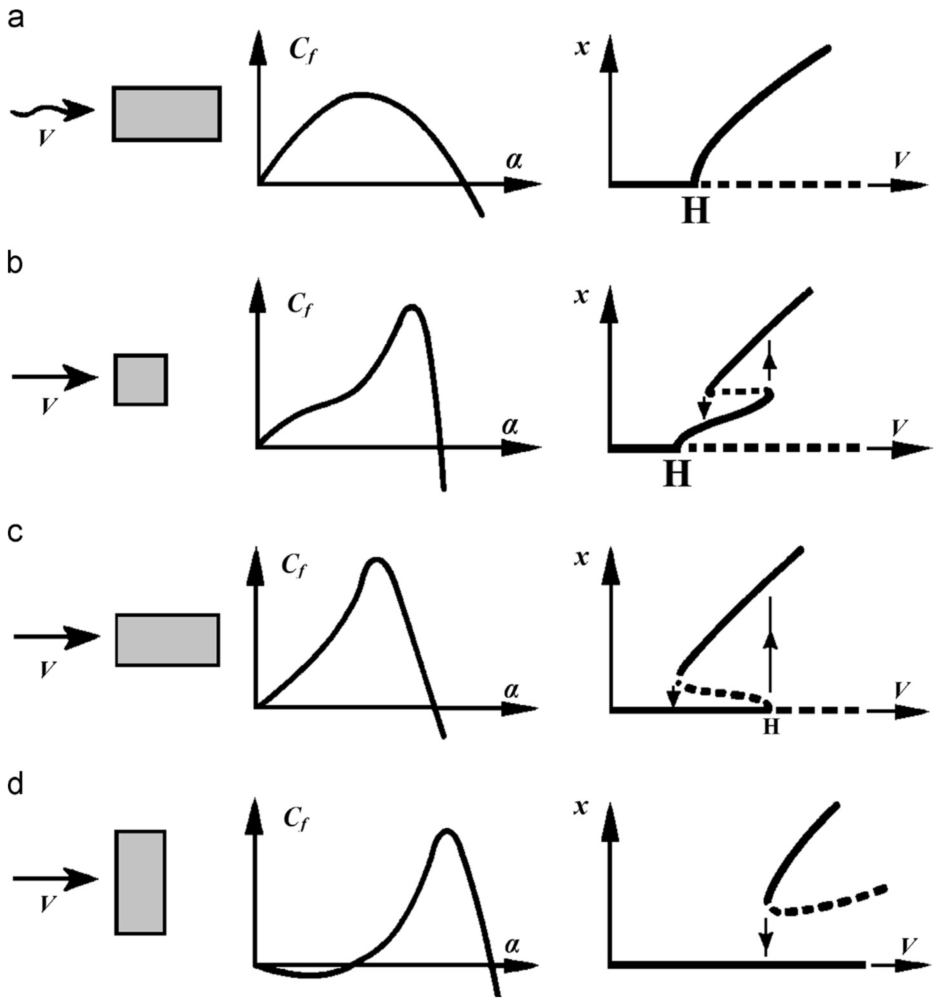

Fig. 1. Various aerodynamic characteristics (first column) and their corresponding dynamic responses (second column) due to Novak [1].

to a large amplitude stable limit cycle (a periodic attractor). The unstable path from H eventually stabilizes at a cyclic fold, giving an overall (dynamic) response akin to the (static) response of many shell-buckling problems.

In the bottom row, (d), a 1:2 rectangle standing across-wind gives no bifurcation from the trivial solution but large amplitude stable and unstable cycles do exist, separated again by a fold.

Some of the most familiar examples of galloping arise with engineering cables [6,7], but we should note that a cable of circular cross-section cannot gallop because the (pure drag) force is in the direction of the resultant wind velocity, and therefore opposes any cable motion. Some cables that can and do gallop are shown in Fig. 2.

Galloping problems can also arise in complete structures, such as tower blocks, and here there can be interactions between the wind-induced vibrations and gravity-induced buckling. A classic case was the high-rise Hancock Tower in Boston [8] which had a lot of such problems in its early days. Window panes started falling out, and eventually all 10,344 had to be replaced (the London Shard has 11,000). Occupants suffered from motion sickness, and tuned mass-dampers had to be fitted. There were still problems, however, when a gravitational instability increased the period of vibration from 12 to 16 s. The final cure was to add 1500 t of diagonal steel bracing, costing $5 million. The tower is still standing today; and still winning architectural prizes for its minimalism!

It is the purpose of this paper to examine the interactions between (Hopf) galloping and (pitch-fork) buckling, remembering that simultaneous failure modes often represent a simplistic, though potentially dangerous, optimal design [9]. We introduce an archetypal model which is non-conservative but autonomous, subjected to time-independent loading by a steadily flowing fluid (air or water). It is designed to exhibit sub-critical bifurcations in both galloping and buckling, both of which will trigger a dynamic jump to a remote stable attractor. When there is more than one candidate attractor, the one onto which the structure settles after the Hopf bifurcation can be *indeterminate* [5,10]. This is due to the two-dimensional spiralling outset (unstable manifold) of the Hopf, which makes the outcome sensitive to infinitesimally small variations in starting conditions or parameters. This indeterminacy forms the focus of our investigation.

2. Archetypal model for combined galloping and buckling

We consider the archetypal model, shown in Fig. 3, that we use to study the nonlinear dynamic interactions between galloping and shell-like buckling. A rigid link is pivoted as shown, and held (nominally) vertical by a long spring of stiffness k which is assumed to remain horizontal throughout and is attached to the mass-less rod at a distance L_2 from the pivot. We introduce an imperfection into the model by supposing that this spring is initially too short by y_0 to hold the unloaded rod exactly vertical. Loaded by the mass m of the grey prism, assumed concentrated at a point on the mass-less rod at a distance L_1 from the pivot, this model will exhibit a sub-critical pitch-fork bifurcation. The only interaction with the wind is (considered to be) through the grey

Please cite this article as: Thompson JMT, Sieber J. Nonlinear dynamic Interactions between flow-induced galloping and shell-like buckling. Int. J. Mech. Sci. (2014), http://dx.doi.org/10.1016/j.ijmecsci.2013.12.007