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An elasticity approach for simply-supported isotropic and orthotropic stiffened plates

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ABSTRACT

Static deflections and natural frequencies of vibrations are obtained for blade-stiffened plates using a three-dimensional model for the plate and a two-dimensional (plane stress) model for the stiffener with simply supported edge/end conditions. These are used as a benchmark for assessing the approach based on the classical hairbrush hypothesis. Results obtained by using the rigorous elasticity model for the plate alone or the stiffener alone are also presented. These results indicate the greater importance of non-classical effects in the analysis of stiffened plates as compared to unstiffened plates.

1. Introduction

The widespread use of stiffened plates as primary load bearing structures is testament to their inherent structural efficiency and to the advent of modern state-of-the-art manufacturing and materials technology. Composite stiffened plates, in particular, provide designers with the opportunity to tailor strength, stiffness and other properties as per structural requirements. This has led to prolific use of such structures in the aerospace, civil, automobile, naval and other high performance industries.

The analysis of stiffened plates can be carried out by different approaches. The orthotropic plate "smeared-out" idealization replaces the plate–stiffener system with an equivalent homogeneous plate with orthotropic properties [1]. While this idealization simplifies the analysis to a great extent, it only provides accurate results when the stiffeners are of uniform size, are closely spaced and their rigidities do not dominate the plate rigidity. Furthermore, this smearing-out of stiffener properties leads to a loss in the discrete nature of the plate–stiffener system and hence in capturing the influence of different geometric parameters on its response.

The plate–beam discrete idealization involves isolating the plate from the beam, modelling them using various simplified theories (usually based on Kirchhoff–Love hypothesis) and maintaining compatibility at the interfaces. However, because of the difficulty in developing general closed-form analytical solutions for this system, increased emphasis was laid on the development of various computer based approximate and numerical schemes using energy principles [2,3], the constraint method [4], BEM and FEM [5–7].

For composite stiffened plates, the effect of shear deformation on the plate behaviour cannot be neglected. This is because the shear stiffness of such materials is small compared to their bending and membrane stiffness ($E_L/G_{LT}=10-50$) unlike for metals where these stiffnesses are comparable ($E/G=2(1+\nu)\simeq 2.6$). With regard to the unstiffened plate, Pagano [8] and Srinivas et al. [9] formulated 3D elasticity solutions for laminates for capturing the shear deformation completely. As an alternative, shear deformation effects may also be accounted for in a 2-D formulation by assuming appropriate displacement fields, as has been illustrated by Carrera [10].

Deb et al. [11] developed an approximate shear deformation theory for stiffened plates based on the Reissner–Mindlin plate theory and Timoshenko beam theory and the smeared-out idealization. Mukherjee et al. [12], Sadek et al. [13], Biswal et al. [14] and Ghosh et al. [15] presented finite elements based on a higher order shear deformation theory (HSDT) for static and vibrational analysis of laminated stiffened plates. Bhar et al. [16] carried out a comparison of the finite element results of composite stiffened plates based on first order shear deformation theory (FSDT) and HSDT. They strongly advocate the use of HSDT over the Classical Plate Theory and even FSDT specially when the panels become thick. Sapountzakis et al. [17] presented an optimized model based on the classical approach, which accounts for the inplane forces and displacements at the interface of the plate and the beam. By comparing their results with a number of finite element models, they bring out the importance of considering the inplane shear

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forces for a more accurate description of the behaviour of the stiffened plate. Qing et al. [18] developed a 3D solution for the free vibrations of stiffened plates based on the variational approach, which uses finite elements to solve state vector equations. The model automatically considered transverse shear deformations and rotary inertia.

In this context, the aim of the current work is to present an analytical elasticity solution for blade stiffened plates wherein the plate is modelled as a 3D solid and the stiffener as a plane stress problem, so that non-classical effects such as transverse shear deformation and rotary inertia are automatically accounted for; all the plate edges as well as the ends of the stiffeners are taken to be simply supported. By comparing this elasticity solution with various approximate models, an attempt will be made to quantify the individual contributions of the plate and beam to the total shear deformation of the structure for different geometric parameters and material properties. Both static deflections and free vibrational frequencies will be studied.

2. Formulation

Consider a rectangular plate of sides a, b and uniform thickness h (Fig. 1) with simply supported edges. The plate is integrally stiffened by a single eccentric rectangular beam of height H and breadth B, attached to only one side, say, the bottom surface of the plate along the central line $y=b/2$. The ends of the stiffener are also taken to be simply supported.

The elasticity solution is formulated by isolating the stiffener from the plate and taking into account the continuity conditions at the interface. The plate is modelled using the equations of 3D elasticity while the stiffener is modelled using a plane stress formulation. With regard to the interface tractions, it is assumed that they remain constant over the width B of the stiffener. In the present work, attention is focused on flexure of the plate symmetrically about the stiffener, and hence the torsional behaviour of the stiffener does not come into picture.

2.1. Analysis of the stiffener

Assuming that the stiffener is specially orthotropic with respect to the x–z coordinates with the plane stress constitutive law

$$\begin{Bmatrix} \sigma_x \\ \sigma_z \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_z \\ \gamma_{xz} \end{Bmatrix}$$

the equations of motion can be written in terms of the displacements u and w along the x and z directions respectively, as

$$Q_{11}u_{,xx}+Q_{55}u_{,zz}+(Q_{12}+Q_{55})w_{,xz}=\rho u_{,tt}$$

$$(Q_{12}+Q_{55})u_{,xz}+Q_{55}w_{,xx},+Q_{22}w_{,zz}=\rho w_{,tt} \tag{1}$$

where ρ is the mass density of the stiffener.

Selection of displacement functions

$$u(x,z,t)=\sum_{m=1}^{\infty} U(z)\cos\left(\frac{m\pi}{a}x\right)e^{i\omega t}$$

$$w(x,z,t)=\sum_{m=1}^{\infty} W(z)\sin\left(\frac{m\pi}{a}x\right)e^{i\omega t}$$

where ω is the natural frequency ensures that the shear diaphragm type simple support conditions

$$\text{at } x=0,\ a;\quad w=0,\quad \sigma_x=0$$

are satisfied *a priori*.

Substitution of the above displacement functions into (1) reduces them to a 4th order system of linear ordinary differential equations in z. Following the standard procedure of seeking solutions for $U(z)$ and $W(z)$ as

$$\begin{Bmatrix} U \\ W \end{Bmatrix} = \begin{Bmatrix} U_0 \\ W_0 \end{Bmatrix} e^{\mathbf{sz}}$$

one gets the auxiliary equation as

$$A's^4+B's^2+C'=0 \tag{2}$$

where

$$A'=Q_{22}Q_{55}$$

$$B'=p^2Q_{12}^2-p^2Q_{11}Q_{22}+2p^2Q_{12}Q_{55}+Q_{22}\rho\omega^2+Q_{55}\rho\omega^2$$

$$C'=p^4Q_{11}Q_{55}-p^2Q_{11}\rho\omega^2-p^2Q_{55}\rho\omega^2+\rho^2\omega^4$$

and $p=m\pi/a$

The nature and multiplicity of the roots of (2) depend on the material properties and the assumed initial value of ω and this dictates the final solution. For example, in the case of real and distinct roots, the final solution is of the form:

$$\begin{Bmatrix} u \\ w \end{Bmatrix} = \sum_{m=1}^{\infty}\left(\sum_{i=1}^{4}\begin{bmatrix} C_{1i} \\ C_{2i} \end{bmatrix} e^{\mathbf{s_i z}}\right)\begin{Bmatrix} \cos\left(\frac{m\pi}{a}x\right) \\ \sin\left(\frac{m\pi}{a}x\right) \end{Bmatrix} e^{i\omega t} \tag{3}$$

Of the 8 constants C_{1i}, C_{2i} (for each harmonic m), only 4 are independent. The inter-relationships are established by substituting

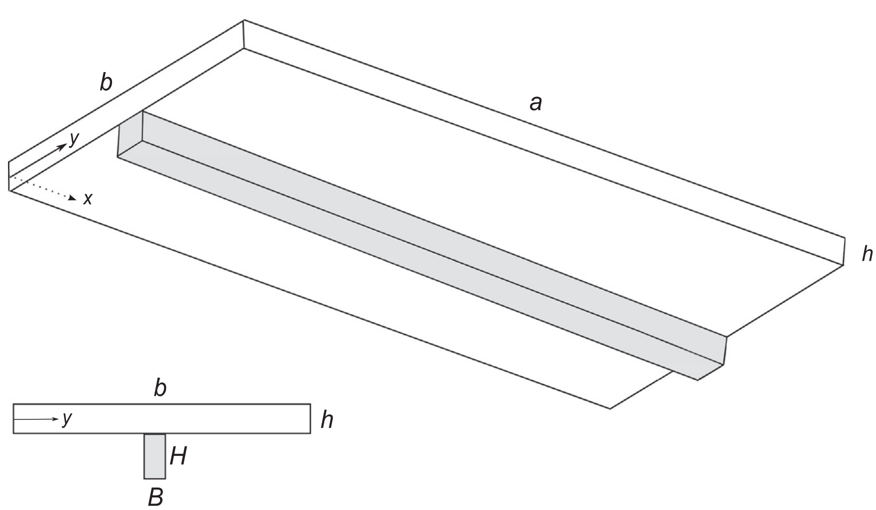

Fig. 1. Eccentrically stiffened plate.