International Journal of Mechanical Sciences 89 (2014) 47–57

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Thermo-elastic constants of cracked symmetric laminates: A refined variational approach

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ARTICLE INFO

Article history:
Received 23 May 2014
Received in revised form
2 August 2014
Accepted 7 August 2014
Available online 17 August 2014

Keywords:
Thermal expansion coefficient
Ply-refinement technique
Intralaminar damage
Stiffness reduction
Levin's theorem
Refined variational approach

ABSTRACT

The present research work is aimed at predicting degradation of laminate stiffness and laminate thermal expansion coefficients (TEC) as a function of crack density for general symmetric laminates containing intralaminar matrix cracks. Therefore, an exact methodology is developed based on Levin's theorem to evaluate the effective thermal expansion coefficients of cracked laminates. Then, an admissible stress field, which satisfies equilibrium equations and all the boundary and continuity conditions, has been considered in conjunction with the principle of minimum complementary energy and a ply refinement technique in order to exactly predict degradation of all thermo-elastic constants of cracked symmetric laminates. Results derived from the developed method for thermo-elastic properties of the cracked laminates showed an excellent agreement with finite element and experimental results available in the literature. Moreover, a comparison has been made between the capability and accuracy of various models in predicting thermo-elastic constants of cracked laminates. It has been shown that the results obtained from the current approach are more accurate and also more consistent with finite element results compared to those obtained from shear-lag approach.

1. Introduction

In fiber reinforced laminated composites, different kinds of micro-damage modes may evolve without leading to final failure [1]. The most common damage modes and the ones examined in this research work are intralaminar cracks (matrix cracks) in layers. Matrix cracks can occur when the laminate is subjected to mechanical or thermal loading and significantly impair the effective thermo-elastic properties of the composite. Matrix cracks and their effects on degradation of material properties have been a focus of extensive research due to their practical importance; see, e.g., reviews [2–5]. In brief, the analytical and numerical models that predict material degradation due to transverse cracking include: shear-lag models [6–8], stress-based variational models [9–12], stress transfer model of McCartney [13], displacement-based variational models [14], models based on considering the role of the crack face relative displacements [15], continuum damage mechanics approach [1,16], discrete damage mechanic models [17], synergistic damage mechanic models [18], multiscale models [19], finite element models [11,20], finite strip models [21], etc.

Of all analytical models, which are focused on developing a representative local stress distribution model between the cracks, the shear-lag and the stress based variational models, are the most widely used approaches [22].

Shear-lag analysis is the simplest way to describe intralaminar cracking in symmetric laminates. This group of models has been used by many authors [22–25]. In the most recent versions of these models [17,20,25,26], where a linear distribution of out-of-plane shear stresses in each layer is assumed, the material properties of the damaged laminate depend exclusively on the crack density and no additional fitting parameters or functions are required. Moreover, the model has the capability of analyzing stiffness and laminate coefficients of thermal expansion degradation for general symmetric laminates with arbitrary stacking sequence. General drawbacks of these models are that the equilibrium equations and the boundary conditions are not satisfied at every single point but only averagely [15,25,26]. However, the most important drawback of these models is that the out-of-plane normal stresses σ_{zz} are neglected in their formulations. This results in the mentioned shear-lag models not to be able to accurately consider the effects of cracked lamina location in the laminate. For example, overall Young's modulus of $[0/90]_s$ and $[90/0]_s$ laminates with identical 90° ply crack density is evaluated with complete correspondence using shear-lag models [25], which is an incorrect result based on both FEM results and experimental observations

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http://dx.doi.org/10.1016/j.ijmecsci.2014.08.008
0020-7403/

[22]. These problems can be more significant where the shear-lag models are used to predict crack evolution. In addition to above drawbacks, since a linear distribution of out-of-plane shear stresses in each layer is assumed, the results obtained from the mentioned methods are not in perfect accordance with refined finite element results.

The variational model, which was introduced by Hashin [9], is also free of any fitting parameters. The approximate stress field derived by this approach satisfies all the necessary equilibrium, continuity and boundary conditions including zero traction on the crack surfaces, but it is based on an oversimplified assumption that the in-plane axial stresses are constant through the thickness coordinate of each layer. Moreover, due to the complexity of the problem, the variational approach has mostly been used for treating either cross-ply laminates [9,10,27–37] or other symmetric laminates reduced to cross-ply by averaging out the off-axis plies [11,37,38]. Recently, Hajikazemi and Sadr [39,40] have overcome this complexity by developing a variational approach, which is capable of analyzing stress field [39] and stiffness reduction [40] of cracked symmetric laminates with arbitrary stacking sequence under general in-plane loading. Nevertheless, following the approach developed by Hashin [9], they also assumed that the in-plane stresses in each layer are independent of the thickness coordinate. Therefore, like many other models based on the principle of minimum complementary energy, it overestimates the stiffness reduction and provides a lower bound for stiffness of a cracked symmetric laminate. It is noteworthy that this lower bound in most cases is not in very good accordance with refined finite element results, especially in the case of laminates with thick layers. Moreover, these models are not able to predict degradation of the laminate coefficients of thermal expansions as they neglect the effects of thermal residual stresses. It is also noted that recently some researchers [11] have tried to increase the accuracy of Hashin's variational method by introducing shape functions with unknown parameters to represent the out-of-plane shear stress distribution across the constraint layer thickness. However, the mentioned method is only capable of modeling the shear modulus reduction of cross laminates. Moreover, the improved stress description in the constraint layer does not seem to be sufficient for accurate predictions of the laminate shear modulus [11].

In the current research work, an attempt has been made to increase the accuracy and versatility of the variational approach for predicting degradation of laminate stiffness and laminate thermal expansion coefficients as a function of crack density for general symmetric laminates containing intralaminar matrix cracks. Thus, in order to predict the laminate thermal expansion coefficients (TEC) of general symmetric laminates, an exact methodology based on Levin's theorem is developed. The method employed is applicable to any cracked laminates, if internal stress fields are known by mechanical loads. It is noted that the special case of the mentioned methodology had been firstly employed by Hashin [29] to predict axial effective thermal expansion coefficients of cracked cross ply laminates. Then, a recently developed admissible stress field [39,40] is considered, which satisfies equilibrium and all the boundary and continuity conditions. This stress field has been used in conjunction with the principle of minimum complementary energy to get the effective thermo-elastic constants of a cracked general symmetric laminate. To increase the accuracy of the model, a ply refinement technique is implemented. With the ply refinement technique, each layer of the laminate is subdivided into plies having the same properties in order that important thickness variations of the stress components could be taken into account. Finally, the problem is reduced to a system of ordinary differential equations that can be solved by standard numerical techniques. The results have been discussed in detail and compared with the finite element solutions recently published by Barbero et al. [20]. As with the shear stiffness modulus, the results have been compared with finite element results reported by Varna and co-workers [41]. The comparisons show an excellent agreement. Also, comparisons to the experimental data [42], the results obtained from the latest versions of shear-lag models [20] and those obtained from a recently developed variational model [11] are presented. It is shown that the current model provides more accurate results compared to those obtained from the shear-lag model and other variational models. It is worth mentioning that the assumed stress field satisfies equilibrium and all the boundary and continuity conditions and here, the principle of minimum complementary energy and an exact methodology based on Levin's theorem are employed to reach the effective stiffness matrix and TEC of cracked laminates. Therefore, the presented methodology, when used in conjunction with ply refinement techniques, can be regarded as a stress based finite super element technique, where accurate solution can be obtained for which continuity of tractions at all element layer interfaces can be guaranteed. Finally, it is noteworthy to mention that recently some researchers have developed models to handle multi-delamination problem of laminated structures with different lay-ups using a layerwise stress approach [43,44] and different shear deformation theories [45,46]. One of the main advantages of these models is that they replace a computationally expensive 3D finite element solution with 2D approaches (equivalent single layer or layerwise). Similar to these approaches, the present variational model can be regarded a basis to develop a 2D stress based layerwise model which can be used to replace a computationally expensive 3D finite element solution in the analysis of cracked or un-cracked symmetric laminates.

2. Evaluation of thermal expansion coefficients (TEC)

In order to obtain the effective TEC of a composite material, assume that a body made of such material is subjected to the homogenous traction boundary conditions as follows:

$$T_i(S) = \sigma_{ij}^{App} n_j \tag{1}$$

where σ_{ij}^{App} is a constant stress tensor and n_j are the components of the outward normal, and to a uniform temperature change of ΔT relative to a uniform reference temperature T_0. Then from average stress theorem, we can write

$$\overline{\sigma_{ij}} = \sigma_{ij}^{App} \tag{2}$$

where the overbar denotes volume average. Thus, the average strain is given by

$$\overline{\varepsilon_{ij}} = S_{ijkl}^{*}\overline{\sigma_{kl}} + \alpha_{ij}^{*}\Delta T \tag{3}$$

where S_{ijkl}^{*} is the effective compliance tensor and α_{ij}^{*} is the effective thermal expansion tensor.

Let the stresses due to mechanical load Eq. (1) only, with no temperature change $\Delta T=0$, be denoted by σ_{ij}^{m} and the stresses due to a unit uniform temperature rise of $\Delta T=1$, with zero boundary traction $T_i(S)=0$, be denoted by σ_{ij}^{t}. Using superposition, the stress field caused by mechanical load and temperature change ΔT in the composite material can be expressed in the following form

$$\sigma_{ij}^{Tot}(X) = \sigma_{ij}^{m}(X) + \Delta T \sigma_{ij}^{t}(X) \tag{4}$$

Levin [47] for the case of two-phase materials and then Rosen and Hashin [48] for the case of multi phase materials have proved a remarkable theorem, which is stated as

$$\int_V \alpha_{ij}(X)\sigma_{ij}^{m}(X)dV = \alpha_{ij}^{*}\overline{\sigma_{ij}}V \tag{5}$$